Exercises in Series

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Convergent or Divergent

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- $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{5n+1}$

Example 1

$$\sum_{n=1}^{\infty} \frac{1}{n + 3^n}$$

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Direct comparison with convergence series $\sum_{n=1}^{\infty} \frac{1}{3^n}$

Example 2

$$\sum_{n=1}^{\infty} \frac{(2n+1)^n}{n^{2n}}$$

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Root test...

$$(|a_n|)^{1/n} = \frac{2n+1}{n^2} \to 0$$

Example 3

$$\sum_{n=1}^{\infty} \frac{(-1)^n n}{n+2}$$

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Diverges by test for divergence

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This is a lot like

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

Does not converge absolutely. Use the Alternating series test.

Example 4

$$\sum_{n=1}^{\infty} \frac{n^2 2^{n-1}}{(-5)^n}$$

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If you're not sure, use the Ratio Test:

$$\lim_{n\to\infty} \frac{(n+1)^2 2^n}{5^{n+1}} \cdot \frac{5^n}{n^2 2^{n-1}} =$$

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Does not converge absolutely. Use the Alternating series test.

Example 5

$$\sum_{n=1}^{\infty} \frac{\sin(2n)}{1 + 2^n}$$

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Direct comparison with $\sum 1/2^n$.

Example 6

$$\sum_{n=1}^{\infty}(-1)^n\frac{n+2}{3n-1}$$

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Diverges by test for divergence (limit of a_n DNE).

Example 7

$$\sum_{n=1}^{\infty} \frac{2^k k!}{(k+2)!}$$

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Check it out with the Ratio Test:

$$\lim_{n\to\infty} \frac{2^{k+1}}{(k+2)(k+3)} \cdot \frac{(k+1)(k+2)}{2^k} = 2 > 1$$