

# Class Handout: 7.3

## “Review” Questions

1. Complete the square:  
(a)  $x^2 - 4x + 6$    (b)  $2x^2 + 3x$    (c)  $x^2 + x + 1$    (d)  $x^2 - 6x + 11$
2. Simplify the following expressions using a triangle  
(a)  $\sin(\tan^{-1}(x))$    (b)  $\cos(\sin^{-1}(x))$    (c)  $\cot(\sin^{-1}(x))$
3. If  $x = 3 \sin(\theta)$ , then find expressions in  $x$  for  $\cos(\theta)$  and  $\sin(2\theta)$ .

## Table that Summarizes Trig Substitutions

Given	Try	With	Restrictions?
$a^2 - u^2$	$u = a \sin(\theta)$	$a^2(1 - \sin^2(\theta)) = a^2 \cos^2(\theta)$	$\theta \in [-\pi/2, \pi/2]$
$a^2 + u^2$	$u = a \tan(\theta)$	$a^2(\tan^2(\theta) + 1) = a^2 \sec^2(\theta)$	$\theta \in (\pi/2, \pi/2)$
$u^2 - a^2$	$u = a \sec(\theta)$	$a^2(\sec^2(\theta) - 1) = a^2 \tan^2(\theta)$	$\theta \in (0, \pi/2) \cup (\pi, 3\pi/2)$

## Exercises

For each of the following, first identify the substitution, then construct the integral in terms of  $\theta$ . Also, draw the right triangle suggested by the substitution.

1.  $\int_0^3 \sqrt{9 - x^2} dx$
2.  $\int \frac{dx}{x^2 \sqrt{x^2 - 9}}$
3.  $\int \sqrt{4x^2 + 20} dx$
4.  $\int \frac{dx}{((x - 3)^2 + 2)^2}$
5.  $\int \frac{t^5}{\sqrt{t^2 + 2}} dt$
6.  $\int_0^1 \sqrt{1 + x^2} dx$
7.  $\int \frac{dx}{x^4 \sqrt{x^2 - 2}}$