Review Problems, Exam 3

1. Does the given sequence or series converge or diverge? If the series converges, is it absolute or conditional? (Careful- we've mixed the sequences and series together!)

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n-\sqrt{n}}$

   (b) $\left\{\frac{n}{1+\sqrt{n}}\right\}$

   (c) $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{n}{n^2+1}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n}{3n+1}\right)$

   (e) $\displaystyle\sum_{n=1}^{\infty} (-6)^{n-1} 5^{1-n}$

   (f) $\left\{\frac{n!}{(n+2)!}\right\}$

   (g) $\displaystyle\sum_{n=1}^{\infty} \frac{1\cdot 3\cdot 5\cdots(2n-1)}{5^n n!}$

   (h) $\displaystyle\sum_{n=2}^{\infty} \frac{3^n+2^n}{6^n}$

   (i) $\left\{\sin\left(\frac{n\pi}{2}\right)\right\}$

   (j) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$

   (k) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2(n)}{n\sqrt{n}}$

   (l) $\displaystyle\sum_{n=1}^{\infty} \frac{(-5)^{2n}}{n^2 9^n}$

   (m) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt{\ln(n)}}$

2. Evaluate the integral or show it diverges:

   (a) $\displaystyle\int_0^1 \frac{x-1}{\sqrt{x}}\,dx$

   (b) $\displaystyle\int_2^{\infty} \frac{1}{x\ln(x)}\,dx$

   (c) $\displaystyle\int_0^{\infty} x^3 e^{-x^4}\,dx$

3. Show that the integral $\displaystyle\int_1^{\infty} \frac{\sin^2(x)}{x^2}\,dx$ converges or diverges. HINT: Do not try to directly compute the antiderivative. Be clear as to your justification.

4. Find the sum of the series (Geometric series)

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{2^{2n}}$

   (b) $\displaystyle\sum_{n=2}^{\infty} \frac{(x-3)^{2n}}{3^n}$

5. Find the radius of convergence. For the last two, include the interval of convergence.

   (a) $\displaystyle\sum \frac{n!x^n}{1\cdot 3\cdot 5\cdots(2n-1)}$

   (b) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^n}{n^2 5^n}$

   (c) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n(x-3)^n}{\sqrt{n+3}}$

6. Use a known template series to find a series for the following:

   (a) $\frac{1}{1+x}$

   (b) $\frac{x}{3-x^2}$

7. True or False, and give a short reason:

   (a) If $\lim_{n\to\infty} a_n = 0$, then the series $\sum a_n$ is convergent.

   (b) If $\sum a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.

   (c) The Ratio Test can be used to determine if a $p-$series is convergent.

(d) If $0 \le a_n \le b_n$ and $\sum b_n$ diverges, then $\sum a_n$ diverges.

(e) If $a_n > 0$ for all n and $\sum a_n$ converges, then $\sum (-1)^n a_n$ converges.

8. Suppose that $\sum_{n=0}^{\infty} c_n (x-1)^n$ converges when $x = 3$ and diverges when $x = -2$.

   (a) What is the largest interval for x on which we can guarantee that the series converges.

   (b) What can be said about the sum: $\sum_{n=0}^{\infty} (-1)^n c_n$

   (c) What can be said about the sum: $\sum_{n=0}^{\infty} c_n 4^n$

9. Since we know that $\int \frac{1}{1+x}\, dx = \ln(1+x)$, find a power series for $\ln(1+x)$ using a geometric series (then integrate it). Also find the radius of convergence.

10. Let $a_n = \frac{2n}{3n+1}$

    (a) Determine whether $\{a_n\}$ is convergent.

    (b) Determine whether $\sum_{n=1}^{\infty} a_n$ is convergent.

11. Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^4}$. Show that the series converges absolutely by using the Integral Test (if appropriate- Check it)

12. Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

    (a) Prove the series converges by using the Alternating Series Test. Be clear about what you have to check for this.

    (b) If we use 99 terms of the series, what is an estimate of the remainder (of the sum)?

13. The terms of a series are defined recursively by the equations:

$$a_1 = 2 \qquad\qquad a_{n+1} = \frac{5n+1}{4n+3} a_n$$

so, for example,

$$a_2 = \frac{6}{7} a_1 = \frac{12}{7}, \qquad a_3 = \frac{11}{11} \cdot a_2 = 1 \cdot \frac{12}{7} = \frac{12}{7}, \cdots$$

Does the series converge or diverge? (Hint: You have enough information to run a convergence test).