# Solutions: Practice with Riemann Sums

1. Write the Riemann sum (left/right as specified) to estimate the area under $f(x)$ on the given interval (This is a definite integral, but we want the Riemann sum).

   (a) $f(x) = x^2$ on $[0, 2]$ using right endpoints.
   Here $\Delta x = \dfrac{2-0}{n} = \dfrac{2}{n}$ and the right endpoints are $x_i = 0 + i\frac{2}{n} = \frac{2i}{n}$. Thus

   $$R_n = \sum_{i=1}^{n} \left(\frac{2i}{n}\right)^2 \cdot \frac{2}{n}$$

   (b) $f(x) = \sqrt{x}$ on $[1, 5]$ using left endpoints.
   Here $\Delta x = \dfrac{5-1}{n} = \dfrac{4}{n}$ and the left endpoints are $x_{i-1} = 1 + (i-1)\dfrac{4}{n}$. Thus

   $$L_n = \sum_{i=1}^{n} \sqrt{1 + \tfrac{4}{n}(i-1)}\ \frac{4}{n}.$$

   (c) $f(x) = e^x$ on $[0, 3]$ using right endpoints.
   Here $\Delta x = \dfrac{3}{n}$ and $x_i = \dfrac{3i}{n}$, so

   $$R_n = \sum_{i=1}^{n} e^{3i/n}\ \frac{3}{n}.$$

2. Convert each Riemann sum into a definite integral.

   (a) $\displaystyle \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \left[\left(2 + \frac{3i}{n}\right)^2 + 2\left(2 + \frac{3i}{n}\right)\right].$

   Note that $\Delta x = \dfrac{3}{n}$ and the $i^{\text{th}}$ right endpoints are $2 + \dfrac{3i}{n}$. The integrand is $f(x) = x^2 + 2x$. Thus one natural expression is

   $$\int_2^5 (x^2 + 2x)\, dx.$$

   We could have made $a = 2$ instead of zero. In that case, the right endpoints are $\dfrac{3i}{n}$, and the function changes to

   $$\int_0^3 \big((2 + 3x)^2 + 2(2 + 3x)\big)\, dx.$$

(b) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{4}{n} \ln\left(1 + \frac{4i}{n}\right)$.

Here $\Delta x = \frac{4}{n}$ and if we let right endpoints be $1 + \frac{4i}{n}$, then

$$\int_1^4 \ln(x)\, dx.$$

(c) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n} \left(3 + \frac{2(i-1)}{n}\right)^3$.

Here $\Delta x = \frac{2}{n}$ and left endpoints $3 + \frac{2(i-1)}{n}$. With $f(x) = x^3$ we obtain

$$\int_3^5 x^3\, dx.$$