Final Exam Review
Calculus II
Sheet 1

1. Prove by induction: $\sum_{i=1}^{n} i^2 = \dfrac{n(n+1)(2n+1)}{6}$

2. State the definition of $\int_a^b f(x)\,dx$.

3. True or False, and give a short reason:

   (a) The Alternating Series Test is sufficient to show that a series is conditionally convergent.

   (b) You can use the Integral Test to show that a series is absolutely convergent.

   (c) Consider $\sum a_n$. If $\lim_{n\to\infty} a_n = 0$, then the sum is said to converge.

   (d) The sequence $a_n = 0.1^n$ converges to $\frac{1}{1-0.1}$

4. Set up an integral for the volume of the solid obtained by rotating the region defined by $y = \sqrt{x-1}$, $y = 0$ and $x = 5$ about the $y-$axis.

5. Write the area under $y = \sqrt[3]{1+x}$, $1 \le x \le 4$ as the limit of a Riemann sum (use **right** endpoints). For the same function, write an integral representing the arc length (do not evaluate the integral).

6. Find the Taylor series for $f(x) = \sqrt{x}$ centered at $a = 4$ (write the first four terms of the series).

7. Find $\frac{dy}{dx}$, if $y = \int_{\cos(x)}^{5x} cos(t^2)\,dt$

8. Let $f(x) = e^x$ on the interval $[0, 2]$. (a) Find the average value of f. (b) Find c that is guaranteed by the Mean Value Theorem for Integrals.

9. Use a template series to find the Maclaurin series for $\int \cos(x^2)\,dx$.

10. Does the series converge (absolute or conditional), or diverge?

    (a) $\sum_{n=1}^{\infty} \dfrac{\cos(n/2)}{n^2+4n}$ (b) $\sum_{n=1}^{\infty} \dfrac{n^2+1}{5^n}$ (c) $\sum_{n=1}^{\infty} \dfrac{3^n n^2}{n!}$

11. Find the interval of convergence:

    (a) $\sum_{n=1}^{\infty} \dfrac{n^2 x^n}{10^n}$ (b) $\sum_{n=1}^{\infty} \dfrac{(3x-2)^n}{n3^n}$ (c) $\sum_{n=1}^{\infty} \dfrac{(-1)^n x^{2n-1}}{(2n-1)!}$

12. Evaluate the integral. You should be able to do these without the table of integrals.

(a) $\displaystyle\int \frac{1}{y^2-4y-12}\,dy$ (c) $\displaystyle\int x^2\cos(3x)\,dx$ (e) $\displaystyle\int \frac{dx}{x\ln(x)}$

(b) $\displaystyle\int \frac{2}{3x+1}+\frac{2x+3}{x^2+9}\,dx$ (d) $\displaystyle\int_{-2}^{2} |x-1|\,dx$ (f) $\displaystyle\int x\sqrt{x-1}\,dx$

13. The velocity function is $v(t) = 3t - 5$, $0 \le t \le 3$ (a) Find the displacement. (b) Find the distance traveled.

14. Use a template series to find the Maclaurin series for the indefinite integral:

$$\int \frac{\mathrm{e}^x - 1}{x}\,dx$$

15. Evaluate the indefinite integral as a power series. What is the radius of convergence?

$$\int \frac{t}{1+t^3}\,dt$$

16. Find the length of the curve

$$y = \int_1^x \sqrt{\sqrt{t}-1}\,dt, \qquad 1 \le x \le 16$$

17. Find an integral that gives the surface area, if the parabola $y = 1 - x^2$, $0 \le x \le 1$, is rotated about the line $y = 2$. (Do not evaluate it).

18. A leaky 10 kg bucket is lifted from the ground to a height of 12 m at a constant speed with a rope that weighs 0.8 kg/m. Initially, the bucket contains 36 kg of water, but the water leaks at a constant rate and finishes draining just as the bucket reaches the 12 m level. Set up the computations you would make to determine how much work is done. You do not have to actually compute the work, just set up the computations and/or integrals needed. You may keep gravity as g m/s^2 , and use the constant σ if you need the density of water (in kg/m^3).