

GROUP WORK 2, SECTION 7.1

Find the Error

It is a beautiful Spring day. You leave your calculus class feeling sad and depressed. You aren't sad because of the class itself. On the contrary, you have just learned an amazing integration technique: Integration by Parts. You aren't sad because it is your birthday. On the contrary, you are still young enough to actually be happy about it. You are sad because you know that every time you learn something really wonderful in calculus, a wild-eyed stranger runs up to you and shows you a "proof" that it is false. Sure enough, as you cross the street, he is waiting on the other side.

"Good morning, Kiddo," he says.

"I just learned integration by parts. Let me have it."

"What do you mean?" he asks.

"Aren't you going to run around telling me that all of math is lies?"

"Well, if you insist," he chuckles... and hands you a piece of paper:

$$\int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx$$
$$u = \frac{1}{\cos x} \qquad dv = \sin x \, dx$$
$$du = \tan x \sec x \, dx \qquad v = -\cos x$$
$$\int \tan x \, dx = uv - \int v \, du$$
$$\int \tan x \, dx = -1 + \int \tan x \, dx$$
$$0 = -1$$

"Hey," you say, "I don't get it! You did everything right this time!"

"Yup!" says the hungry looking stranger.

"But... Zero isn't equal to negative one!"

"Nope!" he says.

You didn't think he could pique your interest again, but he has. Spite him. Find the error in his reasoning.