

GROUP WORK 2, SECTION 7.1

Find the Error (The Sequel)

What a wonderful day! You have survived another encounter with the wild-eyed stranger, demolishing his mischievous pseudo-proof. As you leave his side, you can't resist a taunt.

"Didn't your mother tell you never to forget your constants?" It seemed a better taunt when you were thinking it than it did when you said it.

"Eh?" he says. You come up to him again.

"I was just teasing you. Just pointing out that when doing indefinite integration, those constants should not be forgotten. A simple, silly error, not worthy of you." You look smug. You are the victor.

"Yup. Indefinite integrals always have those pesky constants." For some reason he isn't looking defeated. He is looking crafty.

"Right. Well, I'm going to be going now..."

"Of course, Kiddo, *definite* integrals don't have constants, sure as elephants don't have exoskeletons."

"Yes. Well, I really must be going."

Surprisingly quickly, he snatches the paper out of your hand, and adds to it. This is what it now looks like.

$$\int_{\pi/6}^{\pi/4} \tan x \, dx = \int_{\pi/6}^{\pi/4} \frac{\sin x}{\cos x} \, dx$$
$$u = \frac{1}{\cos x} \qquad dv = \sin x \, dx$$
$$du = \tan x \sec x \, dx \qquad v = -\cos x$$
$$\int_{\pi/6}^{\pi/4} \tan x \, dx = uv - \int_{\pi/6}^{\pi/4} v \, du$$
$$\int_{\pi/6}^{\pi/4} \tan x \, dx = -1 + \int_{\pi/6}^{\pi/4} \tan x \, dx$$
$$0 = -1$$

"No constants missing here! Happy Birthday!" The stranger leaves, singing the "Happy Birthday" song in a minor key. Now there are no constants involved in the argument. But the conclusion is the same: $0 = -1$. Is the stranger right? Has he finally demonstrated that all that you've learned is suspect and contradictory? Or can you, using your best mathematical might, find the error in this new version of his argument?