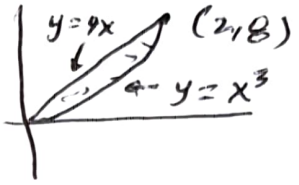
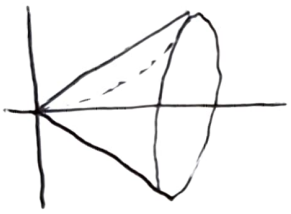
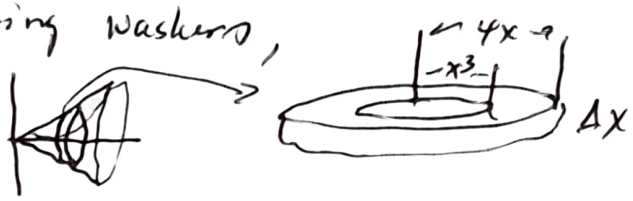
Group Work 1



① Rotate about x-axis

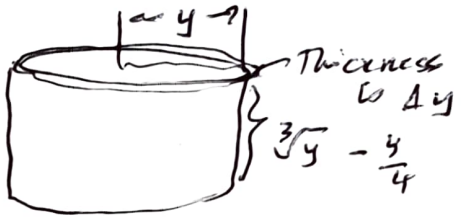


Using Washers,



$$V = \pi \int_0^2 \left((4x)^2 - (x^3)^2 \right) dx$$

Using Shells:



$$V = 2\pi \int_0^8 y \left(\sqrt[3]{y} - \frac{y}{4} \right) dy$$

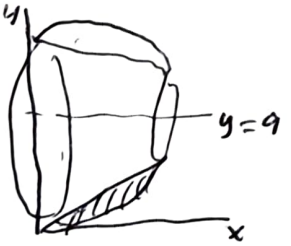
② Rotate about $y = -1$

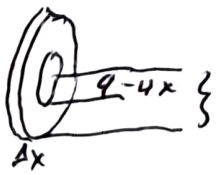
Washers: The radii increase by 1: $V = \pi \int_0^2 \left((4x+1)^2 - (x^3+1)^2 \right) dx$

Shells: Height of shell is same; radius is $1+y$

$$V = 2\pi \int_0^8 (y+1) \left(\sqrt[3]{y} - \frac{y}{4} \right) dy$$

③ Rotate about $y = 9$



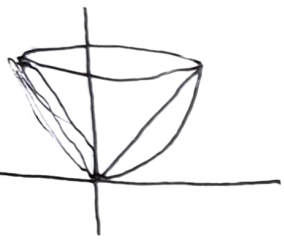


$$V = \pi \int_0^2 \left((9-x^3)^2 - (9-4x)^2 \right) dx$$

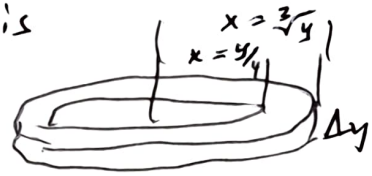
radius is $9-y$
Height is same

$$V = 2\pi \int_0^8 (9-y) \left(\sqrt[3]{y} - \frac{y}{4} \right) dy$$

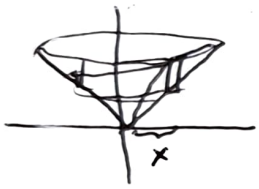
④ Rotate about y-axis



Washer:



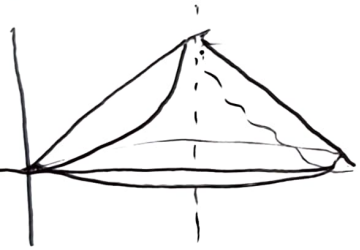
$$V = \pi \int_0^8 \left(\left(\sqrt[3]{y}\right)^2 - \left(\frac{y}{4}\right)^2 \right) dy$$



Shells

$$V = 2\pi \int_0^2 x\left(x^3 - 4x\right) dx$$

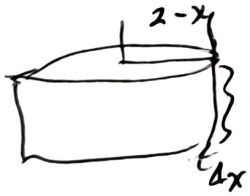
⑤ Rotate about $x = 2$



Washer:

$$V = \pi \int_0^8 \left(\left(2 - \frac{y}{4}\right)^2 - \left(2 - \sqrt[3]{y}\right)^2 \right) dy$$

Shells:



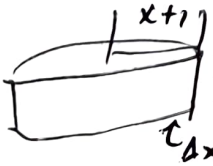
$$V = 2\pi \int_0^2 (2 - x)\left(4x - x^3\right) dx$$

⑥ Rotate about $x = -1$ (Compare to #4)

Washers: Add 1 to each radius:

$$V = \pi \int_0^8 \left(\left(\sqrt[3]{y} + 1\right)^2 - \left(\frac{y}{4} + 1\right)^2 \right) dy$$

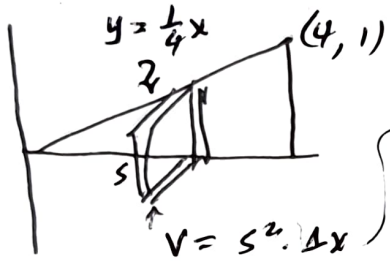
Shells:



$$V = 2\pi \int_0^2 (x + 1)\left(4x - x^3\right) dx$$

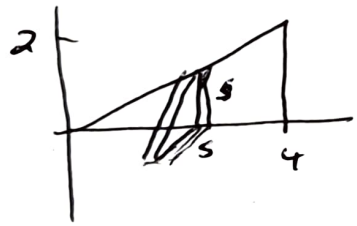
Group Work 2 (Slices)

①



$$V = \int_0^4 \left(\frac{1}{4}x\right)^2 dx$$

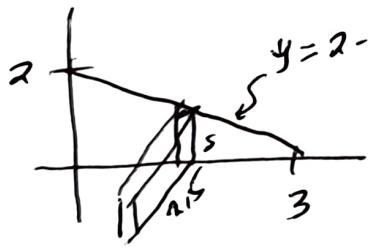
②



$V = \frac{1}{2}s^2 \cdot \Delta x$, so total is

$$V = \int_0^4 \frac{1}{2}\left(\frac{1}{2}x\right)^2 dx$$

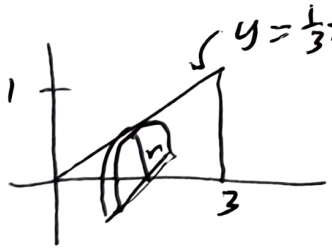
③



$V = s \cdot (2.5) \cdot \Delta x$

$$V = \int_0^3 \left(2 - \frac{2}{3}x\right) \cdot (2.5)\, dx$$

④



$V = \frac{\pi}{2}r^2 \cdot \Delta x$

total: $\frac{\pi}{2}\int_0^3 \left(\frac{1}{3}x\right)^2 \cdot dx$