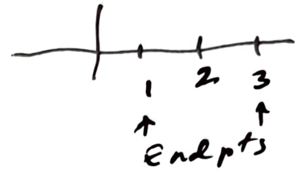
6.1 Handout.

1(a) $\lim_{n\to\infty} \frac{n+1}{n}|x-2| = |x-2| < 1$ $\rho = 1$



If $x=1$:

$\sum_{n=1}^{\infty} n(-1)^n(-1)^n = \sum_{n=1}^{\infty} n$ diverges by Div test

If $x=3$

$\sum_{n=1}^{\infty} n(-1)^n(1)^n = \sum_{n=1}^{\infty} n\cdot(-1)^n$ diverges by div test

$\Rightarrow$ $\rho = 1$, interval is $(1,3)$

(b) $\lim_{n\to\infty} \frac{n^{1/3}}{(n+1)^{1/3}}|x| \to |x| < 1$ so $\rho = 1$

Endpts: $x = \pm 1$

If $x=-1$, $\sum_{n=1}^{\infty} \frac{(-1)^n(-1)^n}{n^{1/3}} = \sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$ divergent p-series

If $x=1$, $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{1/3}}$ conv. by alt series test. (Show details)

oops!

So: $\rho = 1$, interval is $(-1, 1]$

(c) $\lim_{n\to\infty} \frac{n!}{(n+1)!}|x| = 0 \Rightarrow$ converges for all x

$(\rho = \infty)$

(d) $\sum_{n=1}^{\infty} n^n x^n$ Root Test. $\lim_{n\to\infty} \sqrt[n]{n^n|x^n|} = \lim_{n\to\infty} n|x| \nearrow \infty$

for all x except $x=0$.

$\Rightarrow$ $\rho = 0$ (converges at one point only)

e) $\lim\limits_{n\to\infty} \frac{n\,3^n}{(n+1)\,3^{n+1}}\,|x| = \frac{|x|}{3} < 1 \Rightarrow |x| < 3$

$\rho = 3$

Endpts: $x = 3, -3$

If $x = 3$, $\sum\limits_{n=1}^{\infty} \frac{\cancel{3^n}}{n\,\cancel{3^n}} = \sum \frac{1}{n}$ Harmonic series (div)

$x = -3$, $\sum\limits_{n=1}^{\infty} \frac{(-3)^n}{n\,3^n} = \sum\limits_{n=1}^{\infty} \frac{(-1)^n \cancel{3^n}}{n\,\cancel{3^n}} = \sum \frac{(-1)^n}{n}$ Alt Harmonic Series (Converges)

$\rho = 3$, interval is $[-3, 3)$

f) $\lim\limits_{n\to\infty} \frac{4^n \ln(n)}{4^{n+1} \ln(n+1)}\,|x| = \lim\limits_{n\to\infty} \frac{\ln(n)}{\ln(n+1)} \cdot \frac{|x|}{4} = \frac{|x|}{4} < 1$

↑ use l'Hospital

So $\rho = 4$, Endpts are ± 4

At $x = 4$, $\sum\limits_{n=2}^{\infty} \frac{(-1)^n \cdot 4^n}{4^n \ln(n)} = \sum\limits_{n=2}^{\infty} \frac{(-1)^n}{\ln(n)}$

↑ use Alt series test to show convergence.

At $x = -4$, $\sum\limits_{n=2}^{\infty} \frac{(-1)^n (-1)^n 4^n}{4^n \ln(n)} = \sum \frac{1}{\ln(n)}$

We know $x > \ln(x)$, so $\frac{1}{x} < \frac{1}{\ln(x)}$

$\Rightarrow \sum \frac{1}{n}$ ~~converges~~ diverges $\Rightarrow \sum \frac{1}{\ln(n)}$ diverges.

$\rho = 4$, interval is $(-4, 4]$

g) Converges for all x:

$$\lim_{n\to\infty} \frac{(2n+1)!}{(2n+3)!} |x^2| = \lim_{n\to\infty} \frac{1}{(2n+2)(2n+3)} x^2 = 0.$$

h) $$\lim_{n\to\infty} \frac{n^2+1}{(n+1)^2+1} \cdot |x-2| = |x-2| < 1 \Rightarrow \begin{matrix} x=3 \\ x=1 \end{matrix}$$

If $x=3$, $\sum \frac{1}{n^2+1}$ conv,

If $x=1$, $\sum \frac{(-1)^n}{n^2+1}$ conv.

$\Rightarrow$ $\rho = 1$, interval is $[1,3]$

i) $$\lim_{n\to\infty} \frac{2n+1}{2n+3} |x-3| = |x-3| < 1 \qquad \begin{matrix} x=4 \\ x=2 \end{matrix}$$

$x=4$: $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+1}$ – Alt Harmonic Series

$x=2$: $\sum_{n=1}^{\infty} \frac{1}{2n+1}$ Diverges

$\rho = 1$, interval is $(2,4]$

j) $$\lim_{n\to\infty} \frac{3^{n+1}\sqrt{n}}{3^n\sqrt{n+1}} |x-4| = 3|x-4| < 1$$

$|x-4| < 1/3$

If $x = 4+\frac{1}{3}$, $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ div

$x = 4+\frac{1}{3}$, $x = 4-\frac{1}{3}$

$= 13/3$ $\quad$ $= 11/3$

If $x = 4-\frac{1}{3}$, $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ Alt Series Test (conv.)

$\rho = \frac{1}{3}$, interval: $[11/3, \frac{13}{3})$

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(k) $\lim_{n\to\infty} \frac{n+1}{n} \cdot \frac{4^n}{4^{n+1}} |x-1| = \frac{|x-1|}{4} < 1 \Rightarrow |x-1| < 4$

$x = 5$

$x = -3$

$\rho = 4$

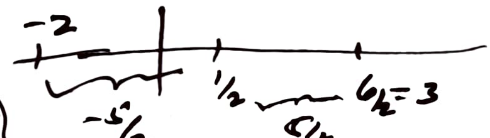
If $x=5$, $\sum \frac{n(4)^n}{4^n}$ div $\Rightarrow (-3, 5)$

If $x=-3$, $\sum n(-1)^n$ div.

l) $\lim_{n\to\infty} \frac{5^n \sqrt{n}}{5^{n+1}\sqrt{n+1}} |2x-1| = \frac{|2x-1|}{5} = \frac{2|x-1/2|}{5} < 1$

$\Rightarrow |x - 1/2| < 5/2$

Endpts: ~~x = 3~~



If $x = 3$: $\sum \frac{5^n}{5^n\sqrt{n}} = \sum \frac{1}{\sqrt{n}}$ div

If $x = -2$, $\sum \frac{(-5)^n}{5^n \sqrt{n}} = \sum \frac{(-1)^n}{\sqrt{n}}$ Alt series test (conv)

$\rho = 5/2$, interval is $[-2, 3)$

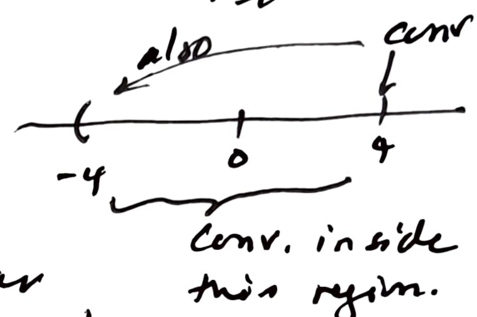
m) $\lim_{n\to\infty} \frac{(n+1)^2}{n^2} \frac{2^n}{2^{n+1}} |x| = \frac{|x|}{2} < 1 \Rightarrow |x| < 2$

If $x = 2 \Rightarrow \sum (-1) n^2$ div.

If $x = -2 \Rightarrow \sum n^2$ div.

$\rho = 2$ interval is $(-2, 2)$

29) $\sum_{n=0}^{\infty} c_n \cdot 4^n$ We can think of $\sum_{n=0}^{\infty} c_n x^n$ at $x=4$



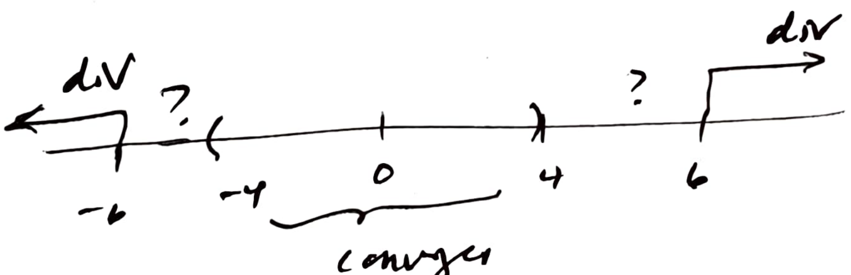
a) $\sum c_n (-4)^n$

↑ Endpt. Not clear if convergent or not.

b) $\sum c_n (-2)^n \Rightarrow x=-2$ is inside the interval

$\Rightarrow$ converges (absolutely)

30) $\sum_{n=0}^{\infty} c_n x^n$ conv. at $x=4$, div. at 6



a) $\sum c_n 8^n \Rightarrow$ div

b) $\sum c_n \cdot 1$ convergent

c) $\sum c_n (-9)^n$ div.

d) $\sum c_n (-3)^n$

↑ inside interval; converges abs.