Hints on the Homework, Section 7.2

You can skip 35-49. Here are hints to the first 34 problems.

1. Reserve $\cos(x)\,dx$ for your du.
2. Reserve $\cos(x)\,dx$ for your du.
3. Reserve either $\sin(x)$ or $\cos(x)$ for your du.
4. Reserve $\sin(x)\,dx$ for your du.
5. Reserve $\cos(\pi x)\,dx$ for your du
6. First do u, du substitution with $u = \sqrt{x}$. Then integrate $\sin^3(u)$ by reserving a $\sin(u)$ for your second substitution.
7. Use the half angle identity
8. Use the half angle identity
9. Square out the half angle identity (and you'll use it again on the result of that).
10. Multiple ways of doing this one. You might break it up as the product of $4\sin^2 t\cos^2 t$ and $\cos^2 t$. Use the formula for $\sin(2\theta)$ for the first, and the half angle identity for the second.
11. Try using the $\sin(2\theta)$ formula, then the half angle.
12. Expand using "FOIL", then use the half angle identity on $\sin^2(\theta)$.
13. Substitute the half angle identity first, then for part of the integral, we'll need to do integration by parts.
14. Do a u, du substitution first- $u = \sin(\theta)$. After that, reserve $\cos(u)\,du$ for a second substitution.
15. Reserve $\cos(\alpha)d\alpha$ for the substitution.
16. If we think about integration by parts, we would keep x in the middle column and $\sin^3(x)$ in the last column. However, we need to integrate that. Do that as a side computation:
$$\int \sin^3(x)\,dx = \cdots$$
17. Rewrite using all sines and cosines, then reserve one $\sin(x)\,dx$ for the substitution.
18. Rewrite using all sines and cosines, then reserve one $\cos(x)\,dx$ for the substitution.
19. Use the identity for $\sin(2x)$, then break up the fraction as a sum of two fractions.
20. Use the identity for $\sin(2x)$, then reserve $\sin(x)\,dx$ for the substitution.
21. Reserve a $\sec(x)\tan(x)\,dx$ for the substitution (substitute $u = \sec(x)$)
22. Reserve $\sec^2(x)\,dx$ for the substitution, substitute $u = \tan(x)$.
23. Use the identity for $tan^2(x) = sec^2(x) - 1$.
24. Use the identity $(1 + \tan^2(x)) = \sec^2(x)$, then reserve that $\sec^2(x)$ for substitution.
25. Reserve $\sec^2(x)\,dx$ for the substitution.
26. Reserve $\sec^2(x)\,dx$ for the substitution.
27. Reserve $\sec^2(x)\,dx$ for the substitution.
28. Reserve $\sec(x)\tan(x)\,dx$ for the substitution.
29. Reserve $\sec(x)\tan(x)\,dx$ for the substitution.
30. Factor as $\tan^2(t)\tan^2(t) = \tan^2(t)(\sec^2(t) - 1)$. To integrate $\tan^2(t)\sec^2(t)$, reserve $\sec^2(t)$. To integrate $\tan^2(t)$, use $\sec^2(t) - 1$ again.
31. We can leave one $\tan(x)$ out, and expand $tan^4(x)$ as $(\sec^2(x) - 1)^2$. A little messy.
32. Write everything in terms of $\sec(x)$. Use our table for $\sec(x), \sec^3(x)$.
33. Integration by parts, middle column x.
34. Reserve $\sin(\phi)d\phi$ for substitution.

55. The average value is:
$$\frac{1}{2\pi}\int_{\pi}^{\pi} \sin^2(x)\cos^3(x)\,dx$$
Reserve one of the cosines for the substitution, $u = \sin(x)$, $du = \cos(x)\,dx$ and rewrite $\cos^2(x) = 1 - \sin^2(x)$.

56. We get the following answers:

(a) $-\frac{1}{2}\cos^2(x) + C_1$

(b) $\frac{1}{2}\sin^2(x) + C_2$

(c) $-\frac{1}{4}\cos(2x) + C_3$

(d) $\frac{1}{2}\sin^2(x) + C_4$

Use the identities $\cos^2(x) = 1 - \sin^2(x)$ and $\cos(2x) = 1 - 2\sin^2(x)$ to see that the answers differ by only a constant.

63. Using washers, the integral should be set up as:

$$V = \int_0^{\pi/4} \pi[(1 - \sin(x))^2 - (1 - \cos(x))^2]\,dx$$

To integrate this, expand and simplify.