Homework Hints, Section 7.8

5. Let $u = x - 2$.

7. Let $u = 3 - 4x$

10. Recall that $\int 2^r \, dr = 2^r / \ln(r)$. (Converges)

13. Break it up- A convenient point is at 0:

$$\int_{-\infty}^{0} x\mathrm{e}^{-x^2}\, dx + \int_{0}^{\infty} x\mathrm{e}^{-x^2}\, dx$$

14. Let $u = \sqrt{x}$. (Converges)

20. Integrate by parts. (Converges)

25. $u = \ln(x)$.

29. Either integrate directly or use $u = x + 2$

31. Break it up- There's a vertical asymptote at $x = 0$.

39. Integrate $z^2 \ln(z)$ by parts. At some point, you might have to take the limit of $t^3(3\ln(t) - 1)$. You might re-write this as $(3\ln(t) - 1)/t^{-3}$ for l'Hospital.

41. Area is $\int_1^{\infty} \mathrm{e}^{-x}\, dx$

49. Note that $\dfrac{x}{x^3+1} < \dfrac{x}{x^3}$ or use limit comparison with $1/x^2$.

50. Note that $\frac{2+\mathrm{e}^{-x}}{x} > \frac{2}{x}$, or use limit comparison with $1/x$ (Diverges).

55. Break it up and deal with the integrals separately

$$\int_0^1 \frac{1}{\sqrt{x}(1+x)}\, dx + \int_1^{\infty} \frac{1}{\sqrt{x}(1+x)}\, dx$$

56. Note that there is a discontinuity at $x = 2$, so break it up as

$$\int_2^3 \frac{dx}{x\sqrt{x^2-4}} + \int_3^{\infty} \frac{dx}{x\sqrt{x^2-4}}$$

Use trig substitution. (Converges to $\pi/4$))

57. We did this one in class.

71. For part (a), note that $\int \mathrm{e}^{-st}\, dt$ is $-\mathrm{e}^{-st}/s$. Something similar for part (b) and (c).