## Exam 3: General Notes

For this exam, we concentrate on Chapter 7 (integration technique, 7.1-7.5 and 7.8), and recall that we had section 6.4 (Work) included.

No calculators will be allowed, but you will be given the list of formulas that we had distributed in class, and constants like density of water and the value of $g$ will be provided.

Please be sure to read the question carefully to see how far into the integral you will need to go. Sometimes I will ask you only to do the set up, sometimes I will ask you to only go as far as the substitution, sometimes you will need to evaluate the integral completely.

As usual, these questions are not meant to be exhaustive, but are meant to give you a good sense of what to expect on the exam. You should also go through the old quizzes and homework.

### Review Questions

1. Write the partial fraction decomposition for each of the following (do not actually solve for the coefficients):

   (a) $\dfrac{3-4x^2}{(2x+1)^3}$ (b) $\dfrac{7x-41}{(x-1)^2(2-x)}$ (c) $\dfrac{x+1}{x^3(x^2-x+10)^2}$

2. Suppose I made the substitution $x = \tan(\theta)$, and after integrating I got the expression $\theta + \sin(2\theta)$. Convert this expression back to $x$.

3. Suppose I made the substitution $t + 2 = \sqrt{3}\sec(\theta)$ and after integrating I got the expression $\cos(2\theta)$. Convert this expression back to $x$.

4. For the following integral, make an appropriate trigonometric substitution and simplify (using trig identities). Leave your answer as an integral in $\theta$.

   (a) $\displaystyle\int \frac{x^2}{(4-x^2)^{3/2}}\,dx$ (b) $\displaystyle\int \frac{x}{x^2+2x+5}\,dx$

5. Show that $\displaystyle\int x f''(x)\,dx = x f'(x) - f(x)$

6. For any spring obeying Hooke's law, show that the work done in stretching a spring a distance of $d$ units (past natural length) is given by $W = \frac{1}{2}kd^2$

7. Set up (do not compute) the integral representing the work done pumping the water over the rim of a tank that is 6 meters long and has a semicircular end of radius 4 meters (the bottom half of a circle). The tank is filled to a depth of 3 meters, and assume the density of water is given by $\sigma$ kg/m$^3$ (where $\sigma$ is approximately constant), and $g = 9.8$m/s$^2$.

8. A 10 lb monkey hangs at the end of a 20 ft chain weighing 1/2 lb per ft. How much work is done by the monkey in climbing up the chain, if the chain is attached to the monkey?

   Hint: You might consider the work using $y$ as the position of the bottom of the chain, so that $0 \le y \le 10$.

9. True or False? (And give a short reason)

   (a) If $f$ is continuous on $[0, \infty)$ and $\int_1^\infty f(x)\,dx$ converges, then so does $\int_0^\infty f(x)\,dx$.

   (b) To find $\int \sin^2(x)\cos^5(x)\,dx$, rewrite the integrand as $\sin^2(x)(1-\sin^2(x))^2\cos(x)$

   (c) Integration by parts is the integral version of the Product Rule for derivatives.

   (d) To find $\int \frac{2x-3}{x^2-3x+5}\,dx$, start by completing the square in the denominator.

   (e) To find $\int \frac{3}{x^2-3x+5}\,dx$, start by completing the square in the denominator.

   (f) To find $\int \frac{3}{x^2-4x+3}\,dx$, start by completing the square in the denominator.

   (g) $u, du$ substitution is the integral version of the Chain Rule.

10. Does the following integral converge or diverge? $\displaystyle\int_1^\infty \frac{2+\sin(x)}{\sqrt{x}}\,dx$

    (Hint: Comparison Theorem)

11. Does the integral converge or diverge? If it converges, evaluate it.

    (a) $\displaystyle\int_0^\infty te^{-st}\,dt$
    ($s$ is a constant- state any conditions on $s$ for the integral to converge.)

    (b) $\displaystyle\int_1^4 \frac{dx}{\sqrt{x-1}}$

    (c) $\displaystyle\int_3^\infty \frac{\ln(x)}{x}\,dx$

    (d) $\displaystyle\int_{-\infty}^\infty \frac{x}{x^2+1}\,dx$

12. Evaluate using any method, unless specified below:

    (a) $\displaystyle\int \frac{4\,dx}{(4+x^2)^{3/2}}$

    (b) $\displaystyle\int \tan^3(x)\sec^2(x)\,dx$

    (c) $\displaystyle\int \frac{3x+2}{x^2+6x+8}\,dx$

    (d) $\displaystyle\int \frac{t^2\cos(t^3-2)}{\sin^2(t^3-2)}\,dt$

    (e) $\displaystyle\int \cos^5(x)\sqrt{\sin(x)}\,dx$

    (f) $\displaystyle\int \frac{x}{x^2+4}\,dx$

    (g) $\displaystyle\int \frac{dx}{\sqrt{1-6x-x^2}}$

    (h) $\displaystyle\int \frac{x-1}{x^2+3}\,dx$

    (i) $\displaystyle\int \sin^2(3t)\,dt$

    (j) $\displaystyle\int \frac{3x-2}{(x^2+2)^2}\,dx$

    (k) $\displaystyle\int \sin^{-1}(x)\,dx$

    (l) $\displaystyle\int x^3\sqrt{x^2+4}\,dx$

    (m) $\displaystyle\int \sqrt{2x-x^2}\,dx$

    (n) $\displaystyle\int \sqrt{t}\ln(t)\,dt$

    (o) $\displaystyle\int \frac{3x-1}{(x+2)(x-3)}\,dx$

    (p) $\displaystyle\int \ln(y^2+9)\,dy$

    (q) $\displaystyle\int \frac{\sin^3(x)}{\cos^4(x)}\,dx$

    (r) $\displaystyle\int e^{-x}\sin(2x)\,dx$

    (s) $\displaystyle\int \frac{w}{\sqrt{w+5}}\,dw$

    (t) $\displaystyle\int y^2e^{-3y}\,dy$

    (u) $\displaystyle\int \frac{y^3+y}{y+1}\,dy$

    (v) $\displaystyle\int x^2e^{2x}\,dx$

    (w) $\displaystyle\int (\ln(x))^2\,dx$

    (x) $\displaystyle\int \frac{2x^3-x^2-4x-13}{x^2-x-2}\,dx$