Review Questions, Exam 1

The following questions are not meant to be exhaustive, so you should also be sure you've looked over your old quizzes and understand the homework.

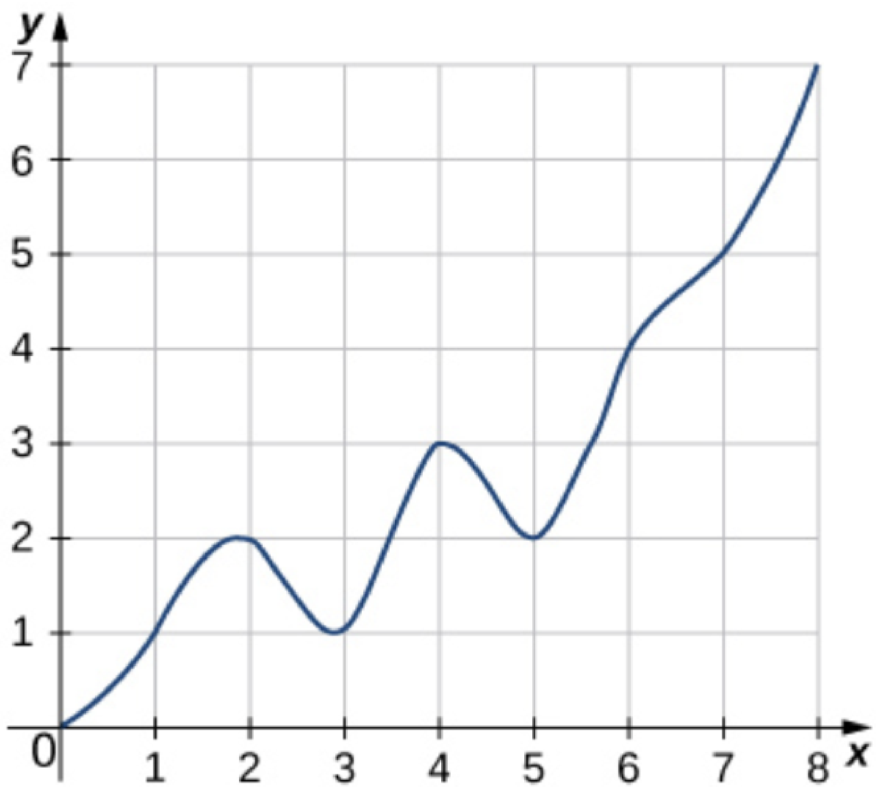
1. Use the Fundamental Theorem of Calculus, Part 1, to differentiate the given integral:

$$\frac{d}{dx}\int_{3x}^{x^2} \frac{\sin(t)}{1+t^2}\,dt$$

2. Evaluate (Hint: Think about what each notation means):

(a) $\int_0^1 \frac{d}{dx}\left(\mathrm{e}^{\tan^{-1}(x)}\right)dx$ (b) $\frac{d}{dx}\int_0^1 \mathrm{e}^{\tan^{-1}(x)}\,dx$ (c) $\frac{d}{dx}\int_0^x \mathrm{e}^{\tan^{-1}(t)}\,dt$

3. The function $y = g(x)$ is plotted below. Estimate by using the Right-Endpoint Approximation with $n = 2$ subintervals on $[0, 8]$. Also, illustrate the corresponding rectangles on the graph below.



4. Given that $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$, find the limit of the sum.

$$\lim_{n\to\infty}\sum_{i=1}^{n}\left(1+\frac{4i}{n}\right)\frac{1}{n}$$

5. For each of the following integrals, use the definition to express each as the limit of an appropriate Riemann sum using right endpoints. (You do not have to evaluate the limit).

   (a) $\int_2^5 x^2\,dx$

   (b) $\int_1^3 1 - 3x\,dx$

6. Find the "c" that is given my the Mean Value Theorem for Integrals, if $f(x) = x^2$ and the interval is $[0, 2]$.

7. True or False (and give a short reason):

   (a) The definite integral $\int_a^b f(x)\,dx$ always equals the area under the graph of $f(x)$ between $x = a$ and $x = b$.

   (b) A Riemann sum uses the idea of dividing an interval $[a, b]$ into subintervals, choosing sample points in each subinterval for heights, and adding up the areas of rectangles.

   (c) If $f(x)$ is an odd function, then $\int_{-a}^a f(x)\,dx = 0$.

   (d) If $\int_a^b f(x)\,dx = 0$, then $f(x) = 0$ for all x in the interval $[a, b]$.

   (e) For any constant c, $\int_a^b c\,dx = c(b - a)$.

   (f) All continuous functions have derivatives.

   (g) All continuous functions have antiderivatives.

8. For each of the following Riemann sums, evaluate the limit by first recognizing it as an appropriate definite integral:

   (a) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{3}{n}\right) \sqrt{1 + \frac{3i}{n}}$ (Find two different integrals for this one!)

   (b) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(2 + 3 \cdot \frac{25i^2}{n^2}\right) \left(\frac{5}{n}\right)$

9. Evaluate the integral, if it exists

   (a) $\int_1^9 \frac{\sqrt{u} - 2u^2}{u}\,du$

   (b) $\int 3^x + \frac{1}{x} + \sec^2(x)\,dx$

   (c) $\int_{-\pi/4}^{\pi/4} \frac{t^4 \tan(t)}{2 + \cos(t)}\,dt$

   (d) $\int_0^3 |x - 2|\,dx$

   (e) $\int \frac{\cos(\ln(x))}{x}\,dx$

   (f) $\int \frac{1}{1 + x^2}\,dx$

   (g) $\int_{-1}^2 \frac{1}{x}\,dx$

   (h) $\int_{-2}^{-1} \frac{1}{x}\,dx$

(i) $\int (1 + \tan(t)) \sec^2(t)\, dt$

(j) $\int x\sqrt{1+x}\, dx$

(k) $\int \frac{y-1}{\sqrt{3y^2 - 6y + 4}}\, dy$

10. Find the derivative of the function:

(a) $F(x) = \int_0^{x^2} \frac{\sqrt{t}}{1+t^2}\, dt$

(b) $y = \int_{\sqrt{x}}^{3x} \frac{e^t}{t}\, dt$

11. Evaluate by recognizing this as a Riemann Sum:

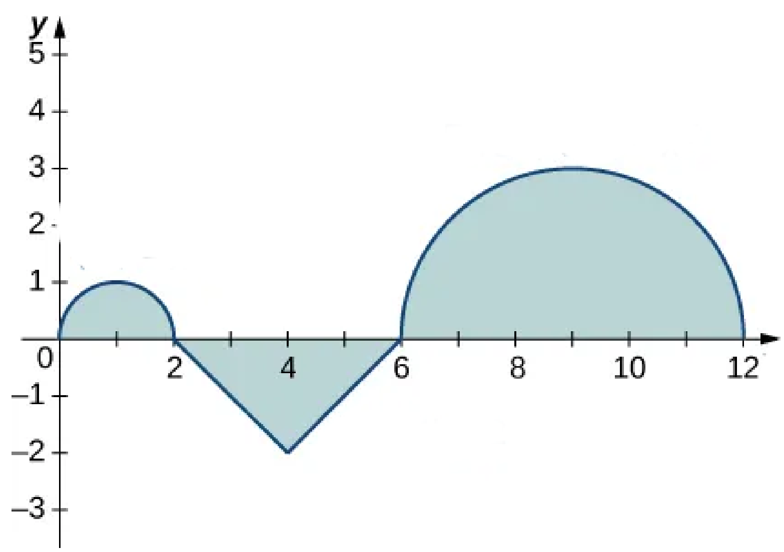
$$\lim_{n\to\infty} \frac{1}{n} \left[\left(\frac{1}{n}\right)^9 + \left(\frac{2}{n}\right)^9 + \left(\frac{3}{n}\right)^9 + \cdots \left(\frac{n}{n}\right)^9 \right]$$

12. Given the graph of $f(t)$ below, let $F(x) = \int_0^x f(t)\, dt$.

(a) Evaluate $F(0)$, $F(2)$, $F(6)$.

(b) Estimate where $F(x)$ is zero the first time for $x > 0$.

(c) Estimate where F attains it's minimum (don't solve for it, try to estimate it).



13. A ball is thrown upward from a height of 3 m at an initial speed of 60 m/s. Acceleration due to gravity is $-9.8\ m/s^2$. Neglecting air resistance, solve for the velocity $v(t)$ and the height $h(t)$ of the ball t seconds after it is thrown and before it returns to the ground. (Hint: Recall the relationship between acceleration, velocity and position (displacement or height).

14. The engine on a boat starts at time $t = 0$ and consumes fuel at the rate $c(t) = 7 - e^{-t}$ litres per hour. How much fuel does it consume for the first 2 hours?