M126
Fall 2016

Quiz 8

Name:

This is a **group quiz.** You may use your text or course notes, and you are encouraged to work together to complete the quiz. You may use your own scratch paper and just write the answers below.

Evaluate the integral:

1. $\int \sin^2(x)\cos^3(x)\,dx$

   IDEA: When one of the powers is odd, reserve one of those for substitution. In this case, we reserve $\cos(x)\,dx$ for u, du substition (so that $u = \sin(x)$). Looking at the integral, we need everything else in terms of the sine function:

   $$\int \sin^2(x)\cos^2(x)\ \cos(x)\,dx = \int \sin^2(x)(1-\sin^2(x))\cos(x)\,dx = \int u^2(1-u^2)\,du = \frac{1}{3}u^3 - \frac{1}{5}u^5 + C$$

   Back-substitute to get the answer:

   $$\frac{1}{3}\sin^3(x) - \frac{1}{5}\sin^5(x) + C$$

2. $\int \frac{dt}{\sqrt{t^2 - 6t + 5}}$

   IDEA: If we complete the square in the denominator, we can use a trig substitution.

   $$\int \frac{dt}{\sqrt{t^2-6t+5}} = \int \frac{dt}{\sqrt{(t^2-6t+9)-4}} = \int \frac{dt}{\sqrt{(t-3)^2-2^2}}$$

   So we substite: $t - 3 = 2\sec(\theta)$ so that:

   $$\int \frac{2\sec(\theta)\tan(\theta)d\theta}{\sqrt{4\sec^2(\theta)-4}} = \int \frac{2\sec(\theta)\tan(\theta)d\theta}{2\tan(\theta)} = \int \sec(\theta)\,d\theta = \ln|\sec(\theta)+\tan(\theta)| + C$$

   With this last integral coming from the table. Use a triangle to back substitute:

   $$\sec(\theta) = \frac{t-3}{2} = \frac{\text{hyp}}{\text{adj}}$$

   so we get:

   $$\ln\left|\frac{t-3}{2} + \frac{\sqrt{t^2-6t+5}}{2}\right| + C$$

3. $\int \frac{x}{(x-2)(x-4)}\,dx$

   IDEA: Partial fractions- $\frac{x}{(x-2)(x-4)} = \frac{A}{x-2} + \frac{B}{x-4}$ Clear denominators to get:

   $$x = A(x-4) + B(x-2) \qquad \text{must be true for all } x$$

   Choose "nice" values of x: For example, if $x = 2$, the equation reduces to $2 = A(-2)$, so that $A = -1$, then if $x = 4$, the equation reduces to $4 = B(2)$, so $B = 2$. Therefore,

   $$\int \frac{x}{(x-2)(x-4)}\,dx = \int \frac{-1}{x-2} + \frac{2}{x-4}\,dx = -\ln|x-2| + 2\ln|x-4| + C$$