Review Problems: Chapter 11

1. What does it mean to say that a series "converges" (I'm looking for the definition; be sure you define any notation you use).

2. Does the given sequence or series converge or diverge?

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n-\sqrt{n}}$

   (b) $\left\{\frac{n}{1+\sqrt{n}}\right\}$

   (c) $\displaystyle\sum_{n=2}^{\infty} \frac{n^2+1}{n^3-1}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{5-2\sqrt{n}}{n^3}$

   (e) $\displaystyle\sum_{n=1}^{\infty} (-6)^{n-1}5^{1-n}$

   (f) $\left\{\frac{n!}{(n+2)!}\right\}$

   (g) $\displaystyle\sum_{n=1}^{\infty} \frac{1\cdot 3\cdot 5\cdots(2n-1)}{5^n n!}$

   (h) $\displaystyle\sum_{n=2}^{\infty} \frac{3^n+2^n}{6^n}$

   (i) $\left\{\sin\left(\frac{n\pi}{2}\right)\right\}$

   (j) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$

   (k) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2(n)}{n\sqrt{n}}$

   (l) $\displaystyle\sum_{n=1}^{\infty} \frac{(-5)^{2n}}{n^2 9^n}$

   (m) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{(n+1)2^n}$

3. Find the sum of the series

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{2^{2n}}$

   (b) $\displaystyle\sum_{n=2}^{\infty} \frac{(x-3)^{2n}}{3^n}$

   (c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n\pi^{2n}}{3^{2n}(2n!)}$

   Hint for 2(c): Does the series look familiar?

4. Find the radius of convergence. For the last two, include the interval of convergence.

   (a) $\displaystyle\sum \frac{n!x^n}{1\cdot 3\cdot 5\cdots(2n-1)}$

   (b) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^n}{n^2 5^n}$

   (c) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n(x-3)^n}{\sqrt{n+3}}$

5. Use a series to evaluate the following limit: $\displaystyle\lim_{x\to 0} \frac{\sin(x)-x}{x^3}$

6. Use a known template series to find a series for the following:

   (a) $\frac{x^2}{1+x}$

   (b) 10^x

   (c) xe^{2x}

   Hint for 5(b): $10^x = e^{\ln(10^x)} = e^{x\ln(10)}$

7. Find the Taylor series for $f(x)$ centered at the given base point:

   (a) x^4-3x^2+1, at $x=1$

   (b) $1/\sqrt{x}$ at $x=9$ (just get the first four non-zero terms of the power series).

   (c) x^{-2} at $x=1$. In this case, find a pattern for the n^{th} coefficient so that you can write the general series. Using this answer, find the radius of convergence.

8. True or False, and give a short reason:

   (a) If $\lim_{n\to\infty} a_n = 0$, then the series $\sum a_n$ is convergent.

(b) If $\sum c_n 6^n$ is convergent, so is $\sum c_n(-2)^n$.

(c) The Ratio Test can be used to determine if a $p-$series is convergent.

(d) If $0 \le a_n \le b_n$ and $\sum b_n$ diverges, then $\sum a_n$ diverges.

(e) $0.9999999\ldots = 1$

(f) If $a_n > 0$ and $\sum a_n$ converges, then $\sum(-1)^n a_n$ converges.

(g) If $f(x) = 2x - x^2 + \frac{1}{3}x^3 - \cdots$ converges for all x, then $f'''(0) = 2$.

9. Suppose that $\sum_{n=0}^{\infty} c_n(x-1)^n$ converges when $x = 3$ and diverges when $x = -2$. What can be said about the convergence or divergence of the following?

   (a) $\sum c_n$ (b) $\sum(-1)^n c_n$ (c) $\sum c_n 3^n$

10. Find the sum: $\sum_{n=2}^{\infty} \frac{n(n-1)}{2^n}$ Hint: Use the geometric series and the derivative

11. Find the Maclaurin series for $\ln(x+1)$ and find the radius of convergence.

12. Find the sum: $3 + \frac{9}{2!} + \frac{27}{3!} + \frac{81}{4!} + \cdots$

13. Use the remainder for the Taylor series to approximate how large the error will be if I use a 3rd order ($n = 3$) Maclaurin series to estimate $\sin(x)$ at $x = 1/2$.

14. Let $a_n = \frac{2n}{3n+1}$

    (a) Determine whether $\{a_n\}$ is convergent.

    (b) Determine whether $\sum_{n=1}^{\infty} a_n$ is convergent.

15. Same as the previous problem, but use $a_n = \dfrac{1+2^n}{3^n}$

16. Compute the sum $1 + a + a^2 + a^3 + a^4 + a^5 + a^6 + a^7$.

    Hint: Your answer should be in terms of a fraction. You might start with s equal to the expression above.

17. Explain the difference between absolute and conditional convergence. Which is "better" and why?