Solutions to Extra Practice in Riemann Sums

1. For each of the following integrals, write the definition using the Riemann sum (and right endpoints), but do not evaluate them:

   (a) $\displaystyle\int_2^5 \sin(3x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} \sin\left(3(2+3i/n)\right)\frac{3}{n}$

   (b) $\displaystyle\int_1^3 \sqrt{1+x}\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} \sqrt{1+(1+2i/n)}\,\frac{2}{n}$

   (c) $\displaystyle\int_0^2 \mathrm{e}^x\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} \mathrm{e}^{2i/n}\frac{2}{n}$

2. For each of the following integrals, write the definition using the Riemann sum, and then evaluate them (MUST use the limit of the Riemann sum for credit, and do not re-write them using the properties of the integral):

   (a) $\displaystyle\int_2^5 x^2\,dx$

$$\lim_{n\to\infty}\sum_{i=1}^{n}\left(2+\frac{3i}{n}\right)^2\frac{3}{n} = \lim_{n\to\infty}\sum_{i=1}^{n}\left(4+\frac{12i}{n}+\frac{9i^2}{n^2}\right)\frac{3}{n} =$$

$$\lim_{n\to\infty}\sum_{i=1}^{n}\left(12+18\cdot\frac{n+1}{n}+\frac{27}{6}\cdot\frac{(n+1)(2n+1)}{n^2}\right) = 39$$

   (b) $\displaystyle\int_1^3 1-3x\,dx$

$$\lim_{n\to\infty}\sum_{i=1}^{n}\left[1-3\left(1+\frac{2i}{n}\right)\right]\frac{2}{n} = \lim_{n\to\infty}\sum_{i=1}^{n}\left[-2-\frac{6i}{n}\right]\frac{2}{n} = \lim_{n\to\infty}-\frac{2}{n}(2n+3(n+1)) = -10$$

   (c) $\displaystyle\int_0^5 1+2x^3\,dx$

   If we simplify the function first,

$$f(5i/n) = 1+2(5i/n)^3 = 1+\frac{250}{n^3}i^3$$

   Using the Riemann sum, if we sum this expression for $i=1..n$, we get

$$\sum_{i=1}^{n} f(5i/n) = n+\frac{250}{n^3}\cdot\frac{n^2(n+1)^2}{4}$$

   Multiply by $5/n$ and take the limit, and we get:

$$5+\frac{625}{2} = \frac{635}{2}$$

3. For each of the following Riemann sums, evaluate the limit by first recognizing it as an appropriate integral:

(a) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{3}{n}\right) \sqrt{1 + \frac{3i}{n}}$ (Find four different integrals for this one!)

Some options:

$$\int_0^3 \sqrt{1+x}\,dx \qquad \int_1^4 \sqrt{1+(x-1)}\,dx = \int_1^4 \sqrt{x}\,dx \qquad \int_2^5 \sqrt{1+(x-2)}\,dx = \int_2^5 \sqrt{x-1}$$

(b) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(2 + 3 \cdot \frac{25i^2}{n^2}\right) \left(\frac{5}{n}\right)$

Some options:

$$\int_0^5 2 + 3x^2\,dx \qquad \int_1^6 2 + 3(x-1)^2\,dx \qquad \int_2^7 2 + 3(x-2)^2\,dx$$

(c) $\lim_{n\to\infty} \sum_{i=1}^{n} \sin\left(3 + \frac{2i}{n}\right) \left(\frac{2}{n}\right)$

Some options:

$$\int_0^2 \sin(3+x)\,dx \qquad \int_3^5 \sin(x)\,dx \text{ etc.}$$