Final Exam Review
Calculus II
Sheet 2

1. True or False, and give a short reason:

   (a) The Ratio Test will not give a conclusive result for $\sum \frac{2n+3}{3n^4+2n^3+3n+5}$

   (b) If $\sum_{n=k}^{\infty} a_n$ converges for some large k, then so will $\sum_{n=1}^{\infty} a_n$.

   (c) If f is continuous on $[0, \infty)$ and $\lim_{x\to\infty} f(x) = 0$, then $\int_0^\infty f(x)\,dx$ converges.

   (d) If f is continuous and $\int_0^9 f(x)\,dx = 4$, then $\int_0^3 xf(x^2)\,dx = 4$.

2. Short Answer:

   (a) Suppose the series $\sum c_n 3^n$ converges. Will $\sum c_n(-2)^n$ also converge? For what values of x will the series $\sum c_n(x-2)^n$ converge?

   (b) If $\sum a_n$, $\sum b_n$ are series with positive terms, and a_n, b_n both go to zero as $n \to \infty$, then what can we conclude if $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$?

   (c) What is the derivative of $\sin^{-1}(x)$? Of $\tan^{-1}(x)$? What is the antiderivative of each?

   (d) Find the sum: $\sum_{n=1}^{\infty} \mathrm{e}^{-2n}$

3. Suppose $h(1) = -2$, $h'(1) = 2$, $h''(1) = 3$, $h(2) = 6$, $h'(2) = 5$, and $h''(2) = 13$, and h'' is continuous. Evaluate $\int_1^2 h''(u)\,du$.

4. Determine a definite integral representing: $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\sqrt{1+\frac{3i}{n}}$ [For extra practice, try writing the integral so that the right endpoint (or bottom bound) must be 5].

5. Evaluate $\int_2^5 (1+2x)\,dx$ by using the definition of the integral (use right endpoints).

6. For each function, find the Taylor series for $f(x)$ centered at the given value of a:

   (a) $f(x) = 1 + x + x^2$ at $a = 2$

   (b) $f(x) = \frac{1}{x}$ at $a = 1$.

7. Find a so that half the area under the curve $y = \frac{1}{x^2}$ lies in the interval $[1, a]$ and half of the area lies in the interval $[a, 4]$.

8. Compute the limit, by using the series for $\sin(x)$: $\lim_{x\to 0} \frac{\sin(x)}{x}$

9. Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by $y = x$, $y = 4x - x^2$, about $x = 7$.

10. Evaluate each of the following:

(a) $\frac{d}{dx}\int_{3x}^{\sin(x)} t^3\, dt.$ (b) $\frac{d}{dx}\int_1^5 x^3\, dx$ (c) $\int_1^5 \frac{d}{dx}x^3\, dx$

11. Converge (absolute or conditional) or Diverge?

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{(n+1)(n+2)}$ (b) $\sum_{n=1}^{\infty} \frac{\sqrt{n^2-1}}{n^3+2n^2+5}$ (c) $\sum_{k=1}^{\infty} \frac{4^k+k}{k!}$

12. Find the interval of convergence.

(a) $\sum_{n=1}^{\infty} n^n x^n$ (b) $\sum_{n=1}^{\infty} \frac{(x+2)^n}{n4^n}$ (c) $\sum_{n=1}^{\infty} \frac{2^n(x-3)^n}{\sqrt{n+3}}$

13. Evaluate:

(a) $\int_0^{\infty} \frac{1}{(x+2)(x+3)}\, dx$ (d) $\int \frac{\tan^{-1}(x)}{1+x^2}\, dx$ (g) $\int e^{-x}\sin(2x)\, dx.$

(b) $\int u(\sqrt{u}+\sqrt[3]{u})\, du$ (e) $\int \frac{1}{\sqrt{x^2-4x}}\, dx$ (h) $\int_0^3 \frac{1}{\sqrt{x}}\, dx$

(c) $\int \frac{x^2}{(4-x^2)^{3/2}}\, dx$ (f) $\int x^4 \ln(x)\, dx$ (i) $\int \sin^2 x \cos^5 x\, dx$

14. A leaky 10-kg bucket is lifted from the ground to a height of 12 meters at a constant speed with a rope has density 0.8 kg/m. Initially the bucket contains 36 kg of water, but the water leaks at a constant rate and finishes draining just as the bucket reaches the 12 meter level. Set up the integral to compute how much work is done (gravity is 9.8 m/s^2):

15. Prove the following by induction:

$$1\cdot 2 + 2\cdot 3 + 3\cdot 4 + \cdots + n\cdot(n+1) = \frac{n(n+1)(n+2)}{3}$$