Hints for Section 7.3

4. You can actually do either $x = \sin(\theta)$ (after that substitution, you can write everything in terms of $u = \cos(\theta)$), or you can try $u = 1 - x^2$ and $du = -2x\,dx$
5. Let $t = \sec(\theta)$, and you should end up with the integral of $\cos^2(\theta)$.
6. Either use $x = 6\sin(\theta)$, and end up integrating just $\sin(\theta)$, or you can also take $u = 36 - x^2$, and $du = -2x\,dx$.
7. Let $x = a \cdot tan(\theta)$, and end up integrating $\cos(\theta)$.
8. Let $t = 4\sec(\theta)$ and end up integrating $\cos(\theta)$. To get your answer back to t, use a triangle!
9. Let $x = 4\tan(\theta)$, and end up with the integral of $\sec(\theta)$.
10. Let $t = \sqrt{2}\tan(\theta)$, and end up with

$$\int \tan^5(\theta)\sec(\theta)\,d\theta = \int \tan^4(\theta)[\tan(\theta)\sec(\theta)\,d\theta]$$

Write $\tan^4(\theta)$ in terms of the secant, and use $u = \sec(\theta)$.

11. Let $2x = \sin(\theta)$. End up integrating $\cos^2(\theta)$.
12. Let $u = \sqrt{5}\sin(\theta)$, and end up integrating $\csc(\theta)$ (use the table from class).
13. Let $x = 3\sec(\theta)$, and end up integrating $\sin^2(\theta)$.
14. Let $x = \tan(\theta)$, end up integrating $\cos^2(\theta)$.
15. Let $x = a \cdot \sin(\theta)$, and end up integrating

$$\int \sin^2(\theta)\cos^2(\theta)\,d\theta = \int \left(\frac{1}{2}\sin(2\theta)\right)^2\,d\theta = \frac{1}{4}\int \frac{1}{2}(1 - \cos(4\theta))\,d\theta$$

16. Let $x = \frac{1}{3}\sec(\theta)$, and end up integrating $\cos^4(\theta)$ (use the table from class).
17. Let $u = x^2 - 7$, so $du = 2x\,dx$
18. Let $ax = b\sec(\theta)$, end up integrating $\csc(\theta)\cot(\theta)$ (the antiderivative is $-\csc(\theta)$)
19. Let $x = \tan(\theta)$, and end up integrating $\csc(\theta) + \sec(\theta)\tan(\theta)$.
20. Let $u = 1 + x^2$, $du = 2x\,dx$
21. Let $x = \frac{3}{5}\sin(\theta)$, and end up integrating $\sin^2(\theta)$.
22. Let $x = \tan(\theta)$, and end up integrating $\sec^3(\theta)$ (Use the table from class).

23. Complete the square so that $5 + 4x - x^2 = 9 - (x-2)^2$. Now we can substitute $x - 2 = 3\sin(\theta)$ and end up integrating $\cos^2(\theta)$.

24. Complete the square so that $t^2 - 6t + 13 = (t-3)^2 + 4$. Now substitute $t - 3 = 2\tan(\theta)$, and end up integrating $\sec(\theta)$.

25. Complete the square so that

$$x^2 + x + 1 = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4} \qquad \Rightarrow \qquad x + \frac{1}{2} = \frac{\sqrt{3}}{2}\tan(\theta)$$

The integral simplifies to integrating $\tan(\theta)\sec(\theta)$ and $\sec(\theta)$.

26. Complete the square so that

$$3 + 4x - 4x^2 = 4 - 4\left(x - \frac{1}{2}\right)^2 = 4\left(1 - \left(x - \frac{1}{2}\right)^2\right) \qquad \Rightarrow \qquad x - \frac{1}{2} = \sin(\theta)$$

We end up integrating $\sec^2(\theta)$, $\tan(\theta)\sec(\theta)$ and $\tan^2(\theta)$.

27. Complete the square so that $x^2 + 2x = (x+1)^2 - 1$, and let $x + 1 = \sec(\theta)$. We end up integrating $\tan^2(\theta)\sec(\theta)$ (write all in terms of $\sec(\theta)$ and use the table.

28. Complete the square so that $x^2 - 2x + 2 = (x-1)^2 + 1$, and let $x - 1 = \tan(\theta)$. We end up integrating $\sin^2(\theta) + 2\sin(\theta)\cos(\theta) + 2\cos^2(\theta)$.

29. Let $u = x^2$ and $du = 2x\,dx$. Then let $u = \sin(\theta)$ (or do the substitution directly by taking $x^2 = \sin(\theta)$, etc. End up integrating $\cos^2(\theta)$.

31(a) Let $x = a\tan(\theta)$, and end up integrating $\sec(\theta)$.

33. First, set up the average value:

$$\frac{1}{6}\int_1^7 \frac{\sqrt{x^2-1}}{x}\,dx$$

Then take $x = \sec(\theta)$, etc. End up integrating $\tan^2(\theta)$.

35. The area of the triangle POQ is

$$\frac{1}{2}(r\cos(\theta))(r\sin(\theta)) = \frac{1}{2}r^2\cos(\theta)\sin(\theta)$$

The area of PQR is

$$\int_{r\cos(\theta)}^{r} \sqrt{r^2 - x^2}\,dx$$

We find that the area of the desired sector is the sum of the area of the triangle and the area using the integral above. Summing these, we get $\frac{1}{2}r^2\theta$.

37. Use disks about the $x-$axis. We end up integrating $1/(x^2+9)^2$, so use $x = 3\tan(\theta)$, and end up integrating $\cos^2(\theta)$.