Homework Hints, Section 7.4

7. Do long division.

8. Do long division.

9. $\dfrac{5x+1}{(2x+1)(x-1)} = \dfrac{A}{2x+1} + \dfrac{B}{x-1}$

10. Integrate $\dfrac{4/9}{y+4} + \dfrac{1/9}{2y-1}$

12. Integrate $\dfrac{2}{x-2} - \dfrac{1}{x-3}$

15. Long division first.

16. Long division first.

18. Integrate $\dfrac{1}{x} - \dfrac{1}{x+1} + \dfrac{1}{x-1}$

20. Integrate $\dfrac{3}{2x+1} - \dfrac{1}{x-2} + \dfrac{2}{(x-2)^2}$

22. Integrate $\dfrac{2}{s} + \dfrac{1}{s^2} - \dfrac{2}{s-1} + \dfrac{1}{(s-1)^2}$

24. Integrate $\dfrac{2}{x} - \dfrac{x+1}{x^2+3}$. NOTE: To integrate the second term, break up the fractions, so that

$$\int \frac{x+1}{x^2+3}\,dx = \int \frac{x}{x^2+3}\,dx + \frac{1}{3}\int \frac{dx}{\left(\frac{x}{\sqrt{3}}\right)^2+1}$$

For the first integral, use $u = x^2$. For the second, use $u = x/\sqrt{3}$.

25. The denominator factors as $(x+1)(x^2+1)$. NOTE: I won't ask you to factor cubics like this on a test or quiz, but it's good practice. In this case, an $x+1$ term factors out of the first two and second two terms.

Once factored, the expansion is the following. Break it up to integrate it:

$$\frac{-2}{x+1} + \frac{2x+2}{x^2+1} = \frac{-2}{x+1} + \frac{2x}{x^2+1} + \frac{1}{x^2+1}$$

26. We could do the full expansion, or notice that:

$$\frac{x^2+x+1}{(x^2+1)^2} = \frac{x^2+1}{(x^2+1)^2} + \frac{x}{(x^2+1)^2}$$

27. We get lucky because a few coefficients are zero! We should get

$$\frac{x}{x^2+1} + \frac{1}{x^2+2}$$

To integrate the second one, factor the 2 from the denominator (Like in the solution to (24) above).

28. We should get, for the partial fractions:

$$\frac{1}{x-1} - \frac{1}{(x-1)^2} - \frac{x-1}{x^2+1}$$

29. We did one similar to this in class: Here, we'll complete the square in the denominator to pull off a u, du substitution.

$$\int \frac{x+4}{x^2+2x+5}\,dx = \int \frac{x+1}{x^2+2x+5}\,dx + \int \frac{3}{(x+1)^2+4}\,dx =$$

$$\frac{1}{2}\int \frac{2x+2}{x^2+2x+5}\,dx + \frac{3}{4}\int \frac{1}{[(x+1)/2]^2+1}\,dx$$

For the first integral, let $u = x^2+2x+5$ so that $du = 2x+2\,dx$ and in the second, let $w = (x+1)/2$ so that $dw = dx/2$ and then:

$$= \frac{1}{2}\int \frac{1}{u}\,du + \frac{3}{2}\int \frac{dw}{w^2+1}$$

$$= \frac{1}{2}\ln|x^2+2x+5| + \frac{3}{2}\tan^{-1}\left(\frac{x+1}{2}\right) + C$$

30. The partial fraction decomposition is

$$\frac{x+1}{x^2+1} + \frac{2-x}{x^2+2}$$

To integrate these, break it up into a sum of four terms:

$$\frac{x}{x^2+1} + \frac{1}{x^2+1} + \frac{2}{x^2+2} - \frac{x}{x^2+2}$$

In the third term, factor 2 from the denominator.

31. (I won't ask you to memorize the difference of cubes formula) The partial fraction decomposition is

$$\frac{1/3}{x-1} - \frac{1}{3}\frac{x+2}{x^2+x+1}$$

To integrate the second term, we would need to complete the square:

$$\frac{x+2}{x^2+x+2} = \frac{x+2}{(x+1/2)^2+3/4} = \frac{x+1/2}{(x+1/2)^2+3/4} + \frac{3/2}{(x+1/2)^2+3/4}$$

Continuing with the second term, we would want to factor the 3/4 out of the denominator.

$$\frac{3/2}{\frac{3}{4}\left[\left(\frac{x+1}{(\sqrt{3}/2)}\right)^2+1\right]} = 2\frac{1}{\left(\frac{x+1}{(\sqrt{3}/2)}\right)^2+1}$$

Now when we integrate this, we just let $u = \frac{x+1}{(\sqrt{3}/2)}$

32. We'll complete the square again in the denominator:

$$\frac{x}{x^2+4x+13} = \frac{x}{(x+2)^2+9} = \frac{(x+2)-2}{(x+2)^2+9} =$$

I'll set this one up so that I have the terms I need to make a u, du substitution, then add whatever is left over onto the end:

$$\frac{1}{2}\,\frac{2x+2}{x^2+4x+13} - 2\cdot\frac{1}{(x+2)^2+9}$$

33. We note that we could factor:

$$x^4+4x^2+3 = (x^2+3)(x^2+1)$$

but, since this is a definite integral, and we have such a nice numerator, we might go ahead and perform u, du directly:

$$\frac{x^3+2x}{x^4+4x^2+3} = \frac{1}{4}\,\frac{4x^3+8x}{x^4+4x^2+3}$$

34. Do long division, then factor the denominator so that the integrand becomes

$$x^2 + \frac{-x^2+x-1}{(x+1)(x^2-x+1} = x^2 - \frac{1}{x-1}$$

35. Kind of long to get the partial fraction constants, but here they are:

$$\frac{1/16}{x} + \frac{-(1/16)x}{x^2+4} + \frac{-(1/4)x}{(x^2+4)^2}$$

Hint: The best way to get the coefficients is to multiply it out, then equate the fourth degree coefficients, then the third degree, then the second and so on.

36. We can actually do u, du substitution without partial fractions for this problem.

37. Kind of long to get the constants; here they are

$$\frac{1}{x^2-4x+6} + \frac{x+1}{(x^2-4x+6)^2}$$

To integrate this, put together one numerator so that you can pull off a u, du substitution. In this case,

$$\frac{1}{x^2-4x+6} + \frac{x-2}{(x^2-4x+6)^2} + 3\,\frac{1}{(x^2-4x+6)^2}$$

The middle one is ready to go- To integrate the first and third expressions, first complete the square. Here they are:

- $\dfrac{1}{x^2-4x+6} = \dfrac{1}{(x-2)^2+2} = \dfrac{1}{2}\,\dfrac{1}{[(x-2)/\sqrt{2}]^2+1}$
  To finish this one, let $u = (x-2)/\sqrt{2}$.
- $3\dfrac{1}{(x-2)^2+2} = \dfrac{3}{4}\,\dfrac{1}{([(x-2)/\sqrt{2}]^2+1)^2}$
  To finish this one, let $\tan(\theta) = \dfrac{x-2}{\sqrt{2}}$, so the expression becomes:
  $$\frac{3}{4}\frac{1}{\sec^4(\theta)}$$
  Remember to include $d\theta$ when you integrate!

38. This one is very similar to 37. The partial fraction expansion gives

$$\frac{x}{x^2+2x+2} + \frac{x-2}{(x^2+2x+2)^2}$$

Now we complete the square: $x^2+2x+2 = (x+1)^2+1$. Rewrite the integrand as a sum of four terms:

$$\frac{x+1}{(x+1)^2+1} - \frac{1}{(x+1)^2+1} + \frac{1}{2}\frac{2x+2}{(x^2+2x+2)^2} - 3\,\frac{1}{((x+1)^2+1)^2}$$

39. Let $u = \sqrt{x+1}$ so that $u^2 = x+1$ and $2u\,du = dx$. The integral then changes to

$$\int \frac{2u^2}{u^2-1}\,du$$

(Then do partial fractions on that, but remember to do long division first!)

53. After integration by parts, we get

$$x\ln(x^2-x+2) - \int \frac{2x^2-x}{x^2-x+2}\,dx$$

Do long division, then partial fractions on the second integral.

57. Complete the square in the denominator.

58. Complete the square in the denominator.

60. Let $t = \tan(x/2)$, then use the results of Exercise 59.