Selected Solutions, Section 5.4

10. Hint: Multiply it out first,

$$\int v(v^2+2)^2\,dv = \int v^5 + 4v^3 + 4v\,dv = \frac{1}{6}v^4 + x^4 + 2v^2 + C$$

12. $\frac{1}{3}x^3 + x + \tan^{-1}(x) + C$.

18. The hint is that $\sin(2x) = 2\sin(x)\cos(x)$. Using that,

$$\int \frac{\sin(2x)}{\sin(x)}\,dx = \int \frac{2\sin(x)\cos(x)}{\sin(x)}\,dx = \int 2\cos(x)\,dx = 2\int \cos(x)\,dx = 2\sin(x) + C$$

19. You could use Wolfram Alpha (on the web), and type:

```
plot sin(x)+(1/4)*x^2+1 and sin(x)+(1/4)*x^2-2
```

29. Hint: Algebraically simplify first!

$$\frac{4+6u}{\sqrt{u}} = 4u^{-1/2} + 6u^{1/2}$$

31. Hint: Multiply through by x first.

33. Recall that $\int 1/x\,dx = \ln|x|$

37. Hint: Rewrite the integrand-

$$\frac{1+\cos^2(\theta)}{\cos^2(\theta)} = \frac{1}{\cos^2(\theta)} + 1 = \sec^2(\theta) + 1$$

45. Hint: Rewrite the integrand to get rid of the absolute value bars. That is,

$$x - 2|x| = \begin{cases} 3x & \text{if } x < 0 \\ -x & \text{if } x > 0 \end{cases}$$

Now, the integral can be broken up:

$$\int_{-1}^{0} \cdots\,dx + \int_0^2 \cdots\,dx$$

49. Hints: The rectangles with which we estimate the area will be horizontal instead of vertical. Therefore, we end up integrating in terms of y instead of x:

$$\int_{y=0}^{y=2} 2y - y^2\,dy = \cdots$$

54. Numerically, that quantity is computed as:

$$100 + n(15) - n(0) = 100 + n(15) - 100 = n(15)$$

and this represents the net change in the bee population in 15 weeks.

61. Hints: You should find that $v(t) = \frac{1}{2}t^2 + 4t + 5$. Remember, to find total distance traveled, we have to compute

$$\int_0^{10} |v(t)|\, dt$$

We find from the quadratic formula that $v(t) = 0$ only for $t = -4 \pm \sqrt{6}$, both of which are negative. Therefore, the velocity is positive for $t > 0$, and we can integrate directly for the total distance:

$$\int_0^{10} \frac{1}{2}t^2 + 4t + 5\, dt = \cdots = \frac{1250}{3}$$

67. From the net change theorem,

$$C(4000) - C(2000) = \int_{2000}^{4000} C'(x)\, dx = \cdots = \$58,000$$

69. From the net change theorem,

$$P(1) - P(0) = \int_0^1 P'(t)\, dt = \int_0^1 1000 \cdot 2^t\, dt = \left.\frac{1000 \cdot 2^t}{\ln(2)}\right|_0^1 = \frac{1000}{ln(2)} \approx 1443$$

Adding in the initial population,

$$P(1) = 1443 + P(0) = 1443 + 4000 = 5443$$