Exam 3: General Notes

For this exam, we concentrate on Chapter 7 (integration technique, 7.1-7.5 and 7.8), and a few applications (6.4 and 8.1). No calculators will be allowed, but you will be given the list of formulas that we had distributed in class.

Please be sure to read the question carefully to see how far into the integral you will need to go. Sometimes I will ask you only to do the set up, sometimes I will ask you to only go as far as the substitution, sometimes you will need to evaluate the integral completely.

As usual, these questions are not meant to be exhaustive, but are meant to give you a better sense of what to expect on the exam. You should also go through the old quizzes and homework.

Review Questions

1. A cross section of a tank of water is the bottom half of a circle of radius 10 ft, and is 50 ft long. Find the work done in pumping the water over the rim of the tank if it filled to a depth of 7 feet (set up the integral only, water weighs 62.5 lbs per cubic feet.) Set up the integral if we were pumping the water up an additional 10 feet up.

2. A heavy rope, 20 meters long, weighs 0.5 kg/m and hangs over a building that is 40 meters tall.

    (a) How much work is done pulling the rope to the top? (Note that the acceleration due to gravity is 9.8m/s^2).

    (b) How much work is done pulling half of the rope to the top? (Hint: It makes sense that it is not half your previous answer, right?)

3. Write the partial fraction decomposition for each of the following (do not actually solve for the coefficients):

    (a) $\dfrac{3 - 4x^2}{(2x + 1)^3}$

    (b) $\dfrac{7x - 41}{(x - 1)^2(2 - x)}$

    (c) $\dfrac{x + 1}{x^3(x^2 - x + 10)^2}$

4. Integrate the following:
$$\int \frac{2x^3 - x^2 - 4x - 13}{x^2 - x - 2}\, dx$$

5. If $x = \tan(\theta)$, show that $\sin(2\theta) = \frac{2x}{1+x^2}$.

6. Find the length of the arc of the curve $y = x^{3/2}$ from the point $(1, 1)$ to $(4, 8)$.

7. Show that $\displaystyle\int x f''(x)\, dx = x f'(x) - f(x)$

8. True or False? (And give a short reason)

(a) To find $\int \sin^2(x)\cos^5(x)\,dx$, rewrite the integrand as $\sin^2(x)(1 - \sin^2(x))^2 \cos(x)$

(b) Integration by parts is the integral version of the Product Rule for derivatives.

(c) To find $\int \frac{2x-3}{x^2-3x+5}\,dx$, start by completing the square in the denominator.

(d) To find $\int \frac{3}{x^2-3x+5}\,dx$, start by completing the square in the denominator.

(e) To find $\int \frac{3}{x^2-4x+3}\,dx$, start by completing the square in the denominator.

(f) u, du substitution is the integral version of the Chain Rule.

9. Does the integral converge or diverge? If it converges, evaluate it.

(a) $\int_0^\infty te^{-st}\,dt$
(s is a constant- state any conditions
on s for the integral to converge.)

(b) $\int_1^4 \frac{dx}{\sqrt{x-1}}$

(c) $\int_3^\infty \frac{\ln(x)}{x}\,dx$

(d) $\int_{-\infty}^\infty \frac{x}{x^2+1}\,dx$

10. Evaluate using any method, unless specified below:

(a) $\int \frac{4\,dx}{(4+x^2)^{3/2}}$

(b) $\int \tan^3(x)\sec^2(x)\,dx$

(c) $\int \frac{3x+2}{x^2+6x+8}$

(d) $\int \frac{t^2 \cos(t^3-2)}{\sin^2(t^3-2)}\,dt$

(e) $\int \cos^5(x)\sqrt{\sin(x)}\,dx$

(f) $\int \frac{x}{x^2+4}\,dx$

(g) $\int \frac{dx}{\sqrt{1-6x-x^2}}$

(h) $\int \frac{x-1}{x^2+3}\,dx$

(i) $\int \sin^2(3t)\,dt$

(j) $\int \frac{3x-2}{(x^2+2)^2}$

(k) $\int \sin^{-1}(x)\,dx$

(l) $\int x^3\sqrt{x^2+4}\,dx$

(m) $\int \sqrt{2x-x^2}\,dx$

(n) $\int \sqrt{t}\ln(t)\,dt$

(o) $\int \frac{3x-1}{(x+2)(x-3)}\,dx$

(p) $\int \ln(y^2+9)\,dy$

(q) $\int \frac{\sin^3(x)}{\cos^4(x)}\,dx$

(r) $\int e^{-x}\sin(2x)\,dx$

(s) $\int \frac{w}{\sqrt{w+5}}\,dw$

(t) $\int y^2 e^{-3y}\,dy$