

## GROUP WORK 2, SECTION 5.5

### Find the Error

It is a beautiful Spring morning. You and your friends are out on the beach. There are volleyball nets up, several open barbecue pits and picnic tables, friendly and interesting people waiting to meet people just like you, and a cabana offering free hot-dogs and broccoli bites. You are all sitting in a circle, practicing integration by substitution. You feel a presence behind you, and see a wild-eyed, hungry-looking stranger staring down.

“What are you kids doing lollygagging around? Don’t you have any work to do?”

“We are practicing integration by substitution,” you explain. “One of us does an integral, and the others check to see if it was done right. It’s fun!”

“Hmm.” he says. Suddenly he gets an evil twinkle in his eyes. “Maybe I had you kids figured all wrong. Mind if I try?”

You are suspicious. When you’ve seen him before, he has been quite unfriendly to both you and the subject of Calculus in general. But maybe he has finally realized how useful mathematics really is. He writes the following in the sand:

The computation of  $\int_{-1}^1 \frac{dx}{1+x^4}$ :

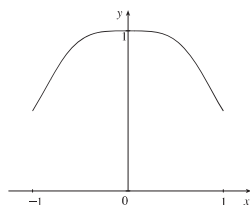
Let  $u = x^2$ , so  $x = u^{1/2}$  and  $dx = \frac{1}{2}u^{-1/2} du$ .

$$\text{Then } \int_{-1}^1 \frac{dx}{1+x^4} = \int_1^1 \left( \frac{1}{1+u^2} \right) \left( \frac{1}{2}u^{-1/2} \right) du.$$

But this is equal to 0 because we are integrating from 1 to 1.

“Now I swear by Eisenhower’s earwax that this problem is correctly done, but maybe you can tell me for sure.” The stranger is looking directly at you when he says this. Your friends are staring, too. You go through it line by line. “It looks okay to me.”

“Funny enough,” he replies, “because last I heard the graph of  $y = \frac{1}{1+x^4}$  looked like this:



And the area under that mugwump is certainly not zero! What do you think of *that*?”

Your friends all smile condescendingly at him, waiting for you to point out the flaw in his “proof”. Will you let them down? Has he shown that the concept of area is useless? Is there really such a thing as a two-by-four? Or can you find what went wrong with the stranger’s reasoning?