

Final Exam Review
Calculus II
Sheet 1

1. Prove by induction: $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$
2. State the definition of $\int_a^b f(x) dx$.
3. True or False, and give a short reason:
 - (a) The Alternating Series Test is sufficient to show that a series is conditionally convergent.
 - (b) You can use the Integral Test to show that a series is absolutely convergent.
 - (c) Consider $\sum a_n$. If $\lim_{n \rightarrow \infty} a_n = 0$, then the sum is said to converge.
 - (d) The sequence $a_n = 0.1^n$ converges to $\frac{1}{1-0.1}$
4. Set up an integral for the volume of the solid obtained by rotating the region defined by $y = \sqrt{x-1}$, $y = 0$ and $x = 5$ about the y -axis.
5. Write the area under $y = \sqrt[3]{1+x}$, $1 \leq x \leq 4$ as the limit of a Riemann sum (use **right** endpoints). For the same function, write an integral representing the arc length (do not evaluate the integral).
6. Find the Taylor series for $f(x) = \sqrt{x}$ centered at $a = 4$ (write the first four terms of the series).
7. Find $\frac{dy}{dx}$, if $y = \int_{\cos(x)}^{5x} \cos(t^2) dt$
8. Let $f(x) = e^x$ on the interval $[0, 2]$. (a) Find the average value of f . (b) Find c that is guaranteed by the Mean Value Theorem for Integrals.
9. Use a template series to find the Maclaurin series for $\int \cos(x^2) dx$.
10. Does the series converge (absolute or conditional), or diverge?
 - (a) $\sum_{n=1}^{\infty} \frac{\cos(n/2)}{n^2 + 4n}$
 - (b) $\sum_{n=1}^{\infty} \frac{n^2 + 1}{5^n}$
 - (c) $\sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$
11. Find the interval of convergence:
 - (a) $\sum_{n=1}^{\infty} \frac{n^2 x^n}{10^n}$
 - (b) $\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n3^n}$
 - (c) $\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n-1}}{(2n-1)!}$

12. Evaluate the integral. You should be able to do these without the table of integrals.

$$\begin{array}{lll} \text{(a)} \int \frac{1}{y^2 - 4y - 12} dy & \text{(c)} \int x^2 \cos(3x) dx & \text{(e)} \int \frac{dx}{x \ln(x)} \\ \text{(b)} \int \frac{2}{3x+1} + \frac{2x+3}{x^2+9} dx & \text{(d)} \int_{-2}^2 |x-1| dx & \text{(f)} \int x\sqrt{x-1} dx \end{array}$$

13. The velocity function is $v(t) = 3t - 5$, $0 \leq t \leq 3$ (a) Find the displacement. (b) Find the distance traveled.

14. Use a template series to find the Maclaurin series for the indefinite integral:

$$\int \frac{e^x - 1}{x} dx$$

15. Evaluate the indefinite integral as a power series. What is the radius of convergence?

$$\int \frac{t}{1+t^3} dt$$

16. Find the length of the curve

$$y = \int_1^x \sqrt{\sqrt{t} - 1} dt, \quad 1 \leq x \leq 16$$

17. Find an integral that gives the surface area, if the parabola $y = 1 - x^2$, $0 \leq x \leq 1$, is rotated about the line $y = 2$. (Do not evaluate it).