Final Exam Review
Calculus II
Sheet 2

1. True or False, and give a short reason:

   (a) The Ratio Test will not give a conclusive result for $\sum \frac{2n+3}{3n^4+2n^3+3n+5}$

   (b) If $\sum_{n=k}^{\infty} a_n$ converges for some large k, then so will $\sum_{n=1}^{\infty} a_n$.

   (c) If f is continuous on $[0,\infty)$ and $\lim_{x\to\infty} f(x) = 0$, then $\int_0^\infty f(x)\,dx$ converges.

   (d) If f is continuous and $\int_0^9 f(x)\,dx = 4$, then $\int_0^3 xf(x^2)\,dx = 4$.

2. Short Answer:

   (a) Suppose the series $\sum c_n 3^n$ converges. Will $\sum c_n(-2)^n$ also converge? For what values of x will the series $\sum c_n(x-2)^n$ converge?

   (b) If $\sum a_n$, $\sum b_n$ are series with positive terms, and a_n, b_n both go to zero as $n \to \infty$, then what can we conclude if $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$?

   (c) What is the derivative of $\sin^{-1}(x)$? Of $\tan^{-1}(x)$? What is the antiderivative of each?

   (d) Find the sum: $\sum_{n=1}^{\infty} \mathrm{e}^{-2n}$

3. Suppose $h(1) = -2$, $h'(1) = 2$, $h''(1) = 3$, $h(2) = 6$, $h'(2) = 5$, and $h''(2) = 13$, and h'' is continuous. Evaluate $\int_1^2 h''(u)\,du$.

4. Determine a definite integral representing: $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\sqrt{1+\frac{3i}{n}}$ [For extra practice, try writing the integral so that the right endpoint (or bottom bound) must be 5].

5. Evaluate $\int_2^5 (1+2x)\,dx$ by using the definition of the integral (use right endpoints).

6. For each function, find the Taylor series for $f(x)$ centered at the given value of a:

   (a) $f(x) = 1 + x + x^2$ at $a = 2$

   (b) $f(x) = \frac{1}{x}$ at $a = 1$.

7. Find a so that half the area under the curve $y = \frac{1}{x^2}$ lies in the interval $[1, a]$ and half of the area lies in the interval $[a, 4]$.

8. Compute the limit, by using the series for $\sin(x)$: $\lim_{x\to 0} \frac{\sin(x)}{x}$

9. Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by $y = x$, $y = 4x - x^2$, about $x = 7$.

10. Evaluate each of the following:

(a) $\dfrac{d}{dx}\displaystyle\int_{3x}^{\sin(x)} t^3\,dt.$ (b) $\dfrac{d}{dx}\displaystyle\int_1^5 x^3\,dx$ (c) $\displaystyle\int_1^5 \dfrac{d}{dx}x^3\,dx$

11. Converge (absolute or conditional) or Diverge?

(a) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n n}{(n+1)(n+2)}$ (b) $\displaystyle\sum_{n=1}^{\infty}\frac{\sqrt{n^2-1}}{n^3+2n^2+5}$ (c) $\displaystyle\sum_{k=1}^{\infty}\frac{4^k+k}{k!}$

12. Find the interval of convergence.

(a) $\displaystyle\sum_{n=1}^{\infty} n^n x^n$ (b) $\displaystyle\sum_{n=1}^{\infty}\frac{(x+2)^n}{n4^n}$ (c) $\displaystyle\sum_{n=1}^{\infty}\frac{2^n(x-3)^n}{\sqrt{n+3}}$

13. Evaluate:

(a) $\displaystyle\int_0^{\infty}\frac{1}{(x+2)(x+3)}\,dx$ (d) $\displaystyle\int\frac{\tan^{-1}(x)}{1+x^2}\,dx$ (g) $\displaystyle\int \mathrm{e}^{-x}\sin(2x)\,dx.$

(b) $\displaystyle\int u(\sqrt{u}+\sqrt[3]{u})\,du$ (e) $\displaystyle\int\frac{1}{\sqrt{x^2-4x}}\,dx$ (h) $\displaystyle\int_0^3\frac{1}{\sqrt{x}}\,dx$

(c) $\displaystyle\int\frac{x^2}{(4-x^2)^{3/2}}\,dx$ (f) $\displaystyle\int x^4\ln(x)\,dx$ (i) $\displaystyle\int\sin^2 x\cos^5 x\,dx$

14. Find a power series for $x^2\ln(5-x)$ by first finding the series for $1/(5-x)$. Give the radius of convergence.

15. Find the Maclaurin series for $x\mathrm{e}^{-x}$ by using a template series.

16. Prove the following by induction:

$$1\cdot 2+2\cdot 3+3\cdot 4+\cdots+n\cdot(n+1)=\frac{n(n+1)(n+2)}{3}$$

17. Find an integral for the surface area of the object obtained by rotating the curve $y=\mathrm{e}^{-x}$ for $0\le x\le 1$, about the $x-$axis. (Do not evaluate)