Final Exam Review
Calculus II
Sheet 3

1. Determine if the series converges (absolute or conditional) or diverges:

   (a) $\sum_{n=1}^{\infty} \frac{(-1)^n \ln(n)}{n}$

   (b) $\sum_{n=1}^{\infty} \frac{e^n}{n!}$

   (c) $\sum_{n=1}^{\infty} \frac{n^3}{e^{n^4}}$

   (d) $\sum_{n=1}^{\infty} 4^{1-2n}$

2. Let $a_n = \frac{n + \ln(n)}{n^2}$.

   (a) Does the sequence $\{a_n\}$ converge or diverge? If it converges, find what it converges to.

   (b) Does the series $\sum_{n=1}^{\infty} a_n$ converge or diverge?

3. A bug is crawling along the graph of the curve $y = 3x + 1$ for x in the interval $[0, t]$. Find the distance the bug has traveled as a function of t.

4. Find the interval of convergence for each of the series:

   (a) $\sum_{n=0}^{\infty} \frac{(2x-3)^n}{n \ln(n)}$

   (b) $\sum_{n=0}^{\infty} \frac{x^n}{n+1}$

   (c) $\sum_{n=0}^{\infty} \frac{3^n x^n}{5^n}$

5. Expand the function $f(x) = \frac{2}{4 - 3x}$ as a power series centered at $x = 0$, and determine the values of x for which the series converges.

6. Evaluate the integral:

   (a) $\int \frac{x^2}{\sqrt{16 - x^2}}\, dx$

   (b) $\int \sin^2(x) \cos^3(x)\, dx$

   (c) $\int x^2 e^{-2x}\, dx$

   (d) $\int \tan^{-1}(x)\, dx$

   (e) $\int \frac{x^2 - x + 1}{x^2 + x}\, dx$

   (f) $\int \frac{dx}{x^2 + 4x - 5}$

   (g) $\int_0^3 |x^2 - 4|\, dx$

   (h) $\int_1^9 \frac{\sqrt{x} - 2x^2}{x}\, dx$

   (i) $\int_{-3}^3 \frac{\sin(x)}{x^2 + 1}\, dx$

7. Evaluate $\int \frac{dx}{x^2 - 1}\, dx$ two ways- Using partial fractions and using trig substitution.

8. Determine if the integral converges or diverges. If it converges, determine what it converges to. $\int_{-\infty}^{9} e^{4x}\, dx$

9. Find a series for $x \tan^{-1}(x^2)$. Hint: You might start with the series for $\tan^{-1}(x)$, which is related to the series for $1/(1 + x^2)$.

10. Consider the region in the first quadrant bounded by the curve $y = 9 - x^2$ with $0 \leq x \leq 3$. Consider the solid obtained by rotating that region about the x axis. Set up two integrals that represent the volume of this solid- One using shells, and one using disks.

11. Same region as before. Set up an integral representing the volume (using any appropriate technique) if the region is revolving about $x = 4$, and then if the region is revolving about $y = -2$.

12. Use differentiation to find a power series for

$$f(x) = \frac{1}{(1+x)^2}$$

13. Use the *definition* of the definite integral (with right endpoints) to calculate the value of $\int_0^2 (x^2 - x)\, dx$.

    (Hint: The formulas for $\sum i^2$ and $\sum i^3$ would be given to you).

14. Find the derivative of the function : $y = \int_{\sqrt{x}}^{x} \frac{e^t}{t}\, dt$

15. Find the c guaranteed by the Mean Value Theorem for Integrals, if $f(x) = 1/x$ on the interval $[1, 3]$. Hint: It has something to do with the average value of f.

16. What is wrong with the following proof:

    Proof by induction that $n + 1 < n$:

    Assume true for $n = k$, so that $k + 1 < k$. We show that this implies $k + 2 < k + 1$:

    Since $k+2 = k+1+1 = (k+1)+1 < k+1$ by induction, then $k+1 < k$ for all positive integers k.