Exam 3: General Notes

For this exam, we concentrate on Chapter 7 (integration technique, 7.1-7.5 and 7.8), and a couple of applications (8.1, 8.2). For Section 8.2 review, see the homework assigned. No calculators will be allowed, but you will be given the list of formulas that we had distributed in class.

Please be sure to read the question carefully to see how far into the integral you will need to go. Sometimes I will ask you only to do the set up, sometimes I will ask you to only go as far as the substitution, sometimes you will need to evaluate the integral completely.

As usual, these questions are not meant to be exhaustive, but are meant to give you a better sense of what to expect on the exam. You should also go through the old quizzes and homework.

Review Questions

1. Write the partial fraction decomposition for each of the following (do not actually solve for the coefficients):

   (a) $\dfrac{3-4x^2}{(2x+1)^3}$ (b) $\dfrac{7x-41}{(x-1)^2(2-x)}$ (c) $\dfrac{x+1}{x^3(x^2-x+10)^2}$

2. Integrate the following:
$$\int \frac{2x^3-x^2-4x-13}{x^2-x-2}\,dx$$

3. Suppose I made the substitution $x=\tan(\theta)$, and after integrating I got the expression $\theta+\sin(2\theta)$. Convert this expression back to x.

4. Suppose I made the substitution $t+2=\sqrt{3}\sec(\theta)$ and after integrating I got the expression $\cos(2\theta)$. Convert this expression back to x.

5. For the following integral, make an appropriate trigonometric substitution and simplify (using trig identities). Leave your answer as an integral in θ.

   (a) $\displaystyle\int \frac{x^2}{(4-x^2)^{3/2}}\,dx$ (b) $\displaystyle\int \frac{x}{x^2+2x+5}\,dx$

6. Find the length of the arc of the curve $y=x^{3/2}$ from the point $(1,1)$ to $(4,8)$.

7. Show that $\displaystyle\int xf''(x)\,dx = xf'(x)-f(x)$

8. True or False? (And give a short reason)

   (a) If f is continuous on $[0,\infty)$ and $\displaystyle\int_1^\infty f(x)\,dx$ converges, then so does $\displaystyle\int_0^\infty f(x)\,dx$.

   (b) To find $\int \sin^2(x)\cos^5(x)\,dx$, rewrite the integrand as $\sin^2(x)(1-\sin^2(x))^2\cos(x)$

   (c) Integration by parts is the integral version of the Product Rule for derivatives.

   (d) To find $\int \frac{2x-3}{x^2-3x+5}\,dx$, start by completing the square in the denominator.

(e) To find $\int \frac{3}{x^2-3x+5}\,dx$, start by completing the square in the denominator.

(f) To find $\int \frac{3}{x^2-4x+3}\,dx$, start by completing the square in the denominator.

(g) u, du substitution is the integral version of the Chain Rule.

9. Does the following integral converge or diverge? $\displaystyle\int_1^\infty \frac{2+\sin(x)}{\sqrt{x}}\,dx$

(Hint: Comparison Theorem)

10. The region under the curve $y = \cos^2(x)$, $0 \le x \le \pi/2$, is rotated about the $x-$axis. Find the volume of the resulting solid.

11. Does the integral converge or diverge? If it converges, evaluate it.

(a) $\displaystyle\int_0^\infty te^{-st}\,dt$
(s is a constant- state any conditions on s for the integral to converge.)

(b) $\displaystyle\int_1^4 \frac{dx}{\sqrt{x-1}}$

(c) $\displaystyle\int_3^\infty \frac{\ln(x)}{x}\,dx$

(d) $\displaystyle\int_{-\infty}^\infty \frac{x}{x^2+1}\,dx$

12. Evaluate using any method, unless specified below:

(a) $\displaystyle\int \frac{4\,dx}{(4+x^2)^{3/2}}$

(b) $\displaystyle\int \tan^3(x)\sec^2(x)\,dx$

(c) $\displaystyle\int \frac{3x+2}{x^2+6x+8}$

(d) $\displaystyle\int \frac{t^2\cos(t^3-2)}{\sin^2(t^3-2)}\,dt$

(e) $\displaystyle\int \cos^5(x)\sqrt{\sin(x)}\,dx$

(f) $\displaystyle\int \frac{x}{x^2+4}\,dx$

(g) $\displaystyle\int \frac{dx}{\sqrt{1-6x-x^2}}$

(h) $\displaystyle\int \frac{x-1}{x^2+3}\,dx$

(i) $\displaystyle\int \sin^2(3t)\,dt$

(j) $\displaystyle\int \frac{3x-2}{(x^2+2)^2}$

(k) $\displaystyle\int \sin^{-1}(x)\,dx$

(l) $\displaystyle\int x^3\sqrt{x^2+4}\,dx$

(m) $\displaystyle\int \sqrt{2x-x^2}\,dx$

(n) $\displaystyle\int \sqrt{t}\ln(t)\,dt$

(o) $\displaystyle\int \frac{3x-1}{(x+2)(x-3)}\,dx$

(p) $\displaystyle\int \ln(y^2+9)\,dy$

(q) $\displaystyle\int \frac{\sin^3(x)}{\cos^4(x)}\,dx$

(r) $\displaystyle\int e^{-x}\sin(2x)\,dx$

(s) $\displaystyle\int \frac{w}{\sqrt{w+5}}\,dw$

(t) $\displaystyle\int y^2e^{-3y}\,dy$