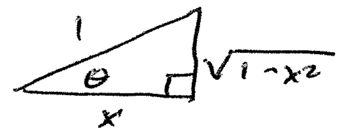
Is it possible to simplify something like:

$$\sin^{-1}(\cos(\theta))?$$

If we let $x = \cos\theta$, we get a triangle:

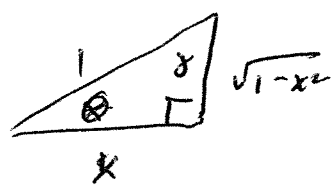


If $\gamma = \sin^{-1}(x)$, then $\sin(\gamma) = x$.

Can we read γ off the triangle?

Angle γ needs to be at the top,

so:



So $\gamma = \frac{\pi}{2} - \theta$

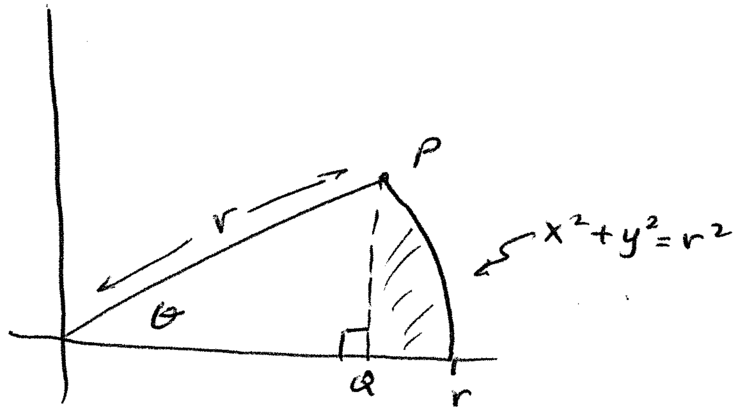
Therefore, $\sin^{-1}(\cos(\theta)) = \frac{\pi}{2} - \theta$

Note: If you know the identity:

$$\cos(\theta) = \sin\left(\frac{\pi}{2} - \theta\right),$$

then take $\sin^{-1}$ of both sides.

#35 "Challenge Problem"



As in class, the area of the triangle is $\frac{1}{2}r^2 \sin\theta \cos\theta$

The area of the shaded region is as given in class,

$$\int_Q^R \sqrt{r^2-x^2}\,dx = \int_{r\cos\theta}^{r} \sqrt{r^2-x^2}\,dx$$

Now, for the trig substitution let $x = r\sin(t)$ (instead of another θ!). Then:

$$dx = r\cos(t)\,dt, \qquad r^2-x^2 = r^2(1-\sin^2(t)) = r^2\cos^2(t)$$

As for the bounds, let's leave them until the end.

Now:

$$\int \sqrt{r^2-x^2}\,dx = \int r^2\cos^2(t)\,dt = \frac{r^2}{2}\int 1+\cos(2t)\,dt$$

$$= \frac{r^2}{2}\left(t + \frac{1}{2}\sin(2t)\right)$$

$$= \frac{r^2}{2}\left(\sin^{-1}\left(\frac{x}{r}\right) + \frac{x}{r}\cdot\frac{\sqrt{r^2-x^2}}{r}\right)$$

$$= \frac{r^2}{2}\sin^{-1}\left(\frac{x}{r}\right) + \frac{1}{2}x\sqrt{r^2-x^2}$$

Bring in the bounds:

$$\left(\frac{r^2}{2} \sin^{-1}\left(\frac{x}{r}\right) + \frac{1}{2} x \sqrt{r^2 - x^2} \right|_{x=a=r\cos\theta}^{x=r}$$

At $x = r$, $\sin^{-1}(1) = \frac{\pi}{2}$ and $\sqrt{r^2 - x^2} = 0$. Evaluate at $x = r\cos\theta$!

$$\frac{r^2}{2}\left(\frac{\pi}{2} + 0\right) - \left(\frac{r^2}{2} \sin^{-1}(\cos\theta) + \frac{1}{2} r\cos\theta \sqrt{r^2 \sin^2\theta}\right)$$

$$\frac{r^2}{2}\frac{\pi}{2} - \left(\frac{r^2}{2}\left(\frac{\pi}{2} - \theta\right) + \frac{1}{2} r^2 \sin\theta\cos\theta\right)$$

$$= \underbrace{\frac{r^2}{2}\theta - \frac{r^2}{2}\sin\theta\cos\theta}_{\text{Circular area}} + \text{Area of triangle.}$$

$$= \frac{r^2\theta}{2} - \frac{r^2}{2}\sin\theta\cos\theta + \frac{r^2}{2}\sin\theta\cos\theta$$

$$= \frac{r^2\theta}{2}$$