Quiz 3 (Section B) Solutions

1. $\displaystyle\sum_{n=1}^{\infty} \frac{n^3 3^n}{n!}$

   SOLUTION: Use the Ratio Test.

$$\lim_{n\to\infty} \frac{3^{n+1}(n+1)^3}{(n+1)!} \cdot \frac{n!}{3^n n^3} = \lim_{n\to\infty} \frac{3(n+1)^3}{(n+1)n^3} = \lim_{n\to\infty} \frac{3(n+1)^2}{n^3} = 0$$

   (The last step was using l'Hospital's rule).

   Therefore, the series converges (absolutely) by the Ratio Test.

2. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{4n-3}{3n+2}$

   SOLUTION: As $n \to \infty$, the terms a_n go to $\pm 4/3$ (so the limit does not exist). Therefore, the series diverges by the Test for Divergence.

3. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \mathrm{e}^{1/n}}{n^3}$

   SOLUTION: A direct comparison might be the fastest. In that case, you would note that $\mathrm{e}^{1/n} < \mathrm{e}^1$ so that:

$$\frac{e^{1/n}}{n^3} < \frac{e}{n^3}$$

   and $\sum \frac{e}{n^3}$ is a convergent p =series. Therefore, the series converges absolutely (by the direct comparison test).

   NOTE: The Ratio Test will give you inconclusive results (because it is so close to a $p-$series).

4. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n)}$

   SOLUTION: First, is this absolutely convergent? No. We can use a comparison test with $\sum 1/n$:

$$\ln(n) < n \quad \Rightarrow \quad \frac{1}{\ln(n)} > \frac{1}{n}$$

   Therefore, the series (as a positive series) diverges by direct comparison.

   Now we can use the Alternating Series Test. Here, $b_n = 1/\ln(n)$.

   - Is the sequence b_n decreasing? Yes, since

$$\frac{1}{\ln(n)} > \frac{1}{\ln(n+1)}$$

   - Is the limit of $b_n = 0$? Yes:

$$\lim_{n\to\infty} \frac{1}{\ln(n)} = 0$$

   Therefore, the series converges conditionally by the Alternating Series Test.