Final Exam Review Calculus II Sheet 2

- 1. True or False, and give a short reason:
 - (a) The Ratio Test will not give a conclusive result for $\sum \frac{2n+3}{3n^4+2n^3+3n+5}$
 - (b) If $\sum_{n=k}^{\infty} a_n$ converges for some large k, then so will $\sum_{n=1}^{\infty} a_n$.
 - (c) If f is continuous on $[0,\infty)$ and $\lim_{x\to\infty} f(x) = 0$, then $\int_0^\infty f(x) dx$ converges.

(d) If f is continuous and
$$\int_0^9 f(x) dx = 4$$
, then $\int_0^3 x f(x^2) dx = 4$.

- 2. Short Answer:
 - (a) Suppose the series $\sum c_n 3^n$ converges. Will $\sum c_n (-2)^n$ also converge? For what values of x will the series $\sum c_n (x-2)^n$ converge?
 - (b) If $\sum a_n$, $\sum b_n$ are series with positive terms, and a_n, b_n both go to zero as $n \to \infty$, then what can we conclude if $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$?
 - (c) What is the derivative of $\sin^{-1}(x)$? Of $\tan^{-1}(x)$? What is the antiderivative of each?
 - (d) Find the sum: $\sum_{n=1}^{\infty} e^{-2n}$
- 3. Suppose h(1) = -2, h'(1) = 2, h''(1) = 3, h(2) = 6, h'(2) = 5, and h''(2) = 13, and h'' is continuous. Evaluate $\int_1^2 h''(u) \, du$.
- 4. Determine a definite integral representing: $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{3}{n}\sqrt{1+\frac{3i}{n}}$ [For extra practice, try writing the integral so that a = 5].
- 5. Evaluate $\int_{2}^{5} (1+2x) dx$ by using the definition of the integral (use right endpoints).
- 6. For each function, find the Taylor series for f(x) centered at the given value of a:

(a)
$$f(x) = 1 + x + x^2$$
 at $a = 2$

- (b) $f(x) = \frac{1}{x}$ at a = 1.
- 7. Find a so that half the area under the curve $y = \frac{1}{x^2}$ lies in the interval [1, a] and half of the area lies in the interval [a, 4].
- 8. Compute the limit, by using the series for $\sin(x)$: $\lim_{x \to 0} \frac{\sin(x)}{x}$
- 9. Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by y = x, $y = 4x x^2$, about x = 7.

10. Evaluate each of the following:

(a)
$$\frac{d}{dx} \int_{3x}^{\sin(x)} t^3 dt.$$
 (b) $\frac{d}{dx} \int_1^5 x^3 dx$ (c) $\int_1^5 \frac{d}{dx} x^3 dx$

11. Converge (absolute or conditional) or Diverge?

(a)
$$\sum_{n=1}^{\infty} \frac{(-1)^n n}{(n+1)(n+2)}$$
 (b) $\sum_{n=1}^{\infty} \frac{\sqrt{n^2 - 1}}{n^3 + 2n^2 + 5}$ (c) $\sum_{k=1}^{\infty} \frac{4^k + k}{k!}$

12. Find the interval of convergence.

(a)
$$\sum_{n=1}^{\infty} n^n x^n$$
 (b) $\sum_{n=1}^{\infty} \frac{(x+2)^n}{n4^n}$ (c) $\sum_{n=1}^{\infty} \frac{2^n (x-3)^n}{\sqrt{n+3}}$

13. Evaluate:

(a)
$$\int_0^\infty \frac{1}{(x+2)(x+3)} dx$$
 (d) $\int \frac{\tan^{-1}(x)}{1+x^2} dx$ (g) $\int e^{-x} \sin(2x) dx$.
(b) $\int u(\sqrt{u} + \sqrt[3]{u}) du$ (e) $\int \frac{1}{\sqrt{x^2 - 4x}} dx$ (h) $\int_0^3 \frac{1}{\sqrt{x}} dx$
(c) $\int \frac{x^2}{(4-x^2)^{3/2}} dx$ (f) $\int x^4 \ln(x) dx$ (i) $\int \sin^2 x \cos^5 x dx$

- 14. Find a power series for $x^2 \ln(5-x)$ by first finding the series for 1/(5-x). Give the radius of convergence.
- 15. Find the Maclaurin series for xe^{-x} by using a template series.
- 16. Prove the following by induction:

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \dots + n \cdot (n+1) = \frac{n(n+1)(n+2)}{3}$$

- 17. Find an integral for the surface area of the object obtained by rotating the curve $y = e^{-x}$ for $0 \le x \le 1$, about the x-axis. (Do not evaluate)
- 18. A bucket weighing 100 lbs is filled with sand weighing 500 lbs. A crane lifts the bucket from the ground to a point 80 ft in the air at a rate of 2 ft/s, but sand is leaking out of a hole at a rate of 3 lbs/s. Neglecting friction and the weight of the cable, find an expression (integral) that will compute the amount of work being done.