Final Exam Review
Calculus II
Sheet 3

1. Determine if the series converges (absolute or conditional) or diverges:

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \ln(n)}{n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n!}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{e^{n^4}}$

   (d) $\displaystyle\sum_{n=1}^{\infty} 4^{1-2n}$

2. Let $a_n = \dfrac{n + \ln(n)}{n^2}$.

   (a) Does the sequence $\{a_n\}$ converge or diverge? If it converges, find what it converges to.

   (b) Does the series $\displaystyle\sum_{n=1}^{\infty} a_n$ converge or diverge?

3. A bug is crawling along the graph of the curve $y = 3x + 1$ for x in the interval $[0, t]$. Find the distance the bug has traveled as a function of t.

4. Find the interval of convergence for each of the series:

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(2x-3)^n}{n \ln(n)}$

   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n+1}$

   (c) $\displaystyle\sum_{n=0}^{\infty} \frac{3^n x^n}{5^n}$

5. Expand the function $f(x) = \dfrac{2}{4-3x}$ as a power series centered at $x = 0$, and determine the values of x for which the series converges.

6. Evaluate the integral:

   (a) $\displaystyle\int \frac{x^2}{\sqrt{16-x^2}}\,dx$

   (b) $\displaystyle\int \sin^2(x)\cos^3(x)\,dx$

   (c) $\displaystyle\int x^2 e^{-2x}\,dx$

   (d) $\displaystyle\int \tan^{-1}(x)\,dx$

   (e) $\displaystyle\int \frac{x^2-x+1}{x^2+x}\,dx$

   (f) $\displaystyle\int \frac{dx}{x^2+4x-5}$

   (g) $\displaystyle\int_0^3 |x^2-4|\,dx$

   (h) $\displaystyle\int_1^9 \frac{\sqrt{x}-2x^2}{x}\,dx$

   (i) $\displaystyle\int_{-3}^3 \frac{\sin(x)}{x^2+1}\,dx$

7. Evaluate $\displaystyle\int \frac{dx}{x^2-1}\,dx$ two ways- Using partial fractions and using trig substitution.

8. Determine if the integral converges or diverges. If it converges, determine what it converges to. $\displaystyle\int_{-\infty}^{9} e^{4x}\,dx$

9. Find a series for $x\tan^{-1}(x^2)$. Hint: You might start with the series for $\tan^{-1}(x)$, which is related to the series for $1/(1+x^2)$.

10. Consider the region in the first quadrant bounded by the curve $y = 9 - x^2$ with $0 \leq x \leq 3$. Consider the solid obtained by rotating that region about the x axis. Set up two integrals that represent the volume of this solid- One using shells, and one using disks.

11. Same region as before. Set up an integral representing the volume (using any appropriate technique) if the region is revolving about $x = 4$, and then if the region is revolving about $y = -2$.

12. Use differentiation to find a power series for

$$f(x) = \frac{1}{(1+x)^2}$$

13. Use the *definition* of the definite integral (with right endpoints) to calculate the value of $\int_0^2 (x^2 - x)\,dx$.

    (Hint: The formulas for $\sum i^2$ and $\sum i^3$ would be given to you).

14. Find the derivative of the function : $y = \int_{\sqrt{x}}^{x} \frac{e^t}{t}\,dt$

15. Find the c guaranteed by the Mean Value Theorem for Integrals, if $f(x) = 1/x$ on the interval $[1, 3]$. Hint: It has something to do with the average value of f.

16. What is wrong with the following proof:

    Proof by induction that $n + 1 < n$:

    Assume true for $n = k$, so that $k + 1 < k$. We show that this implies $k + 2 < k + 1$:

    Since $k+2 = k+1+1 = (k+1)+1 < k+1$ by induction, then $k+1 < k$ for all positive integers k.

17. If the natural length of a spring is 0.2 m, and if it takes a force of 12 N to keep it extended 0.04 meters, find the work done in stretching the spring from its natural length to 0.3 m. You may use leave gravity as g if needed.

18. A tank has a cross sectional area that is a right triangle, with a height of 3 and base of 4 meters. The tank is 10 meters long and full of water. Give an integral that would compute the amount of work involved in pumping the tank dry (the water goes out the top). If needed, use g for acceleration due to gravity (in m/s^2) and σ for the density of water (in kg/m^3).