INSTRUCTIONS: Whenever we evaluate a definite integral, assume the instructions say: "Evaluate the integral, **if it exists**". When evaluating an indefinite integral, you may assume that the function is continuous on its domain.

1. Set up an integral for the volume of the solid obtained by rotating the region defined by $y = \sqrt{x-1}$, $y = 0$ and $x = 5$ about the $y-$axis.

2. Write the area under $y = \sqrt[3]{x}$, $0 \leq x \leq 8$ as the limit of a Riemann sum (use right endpoints).

3. Find the volume of the solid obtained by rotating the region bounded by: $y = \frac{1}{x}$, $y = 0$, $x = 1$, $x = 3$ about $y = -1$.

4. The integral $\pi \int_2^5 y\ dy$ represented the volume of a solid. Describe the solid.

5. Write the appropriate partial fraction expansion for the following expression (do not solve for the constants):
$$\frac{1+x}{(x-1)^2(x^2+1)}$$

6. Let R be the region in the first quadrant bounded by $y = x^3$ and $y = 2x - x^2$. Calculate: (a) The area of R, (b) Volume obtained by rotating R about the $x-$axis (c) Volume obtained by rotating R about the $y-$axis

7. Determine a region whose area is equal to the following limit (do not evaluate the limit):
$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\sqrt{1+\frac{3i}{n}}$$

8. What is the derivative of $\sin^{-1}(x)$? $\tan^{-1}(x)$? What are their antiderivatives?

9. What is the derivative of e^{-2x}? The antiderivative of e^{-3x}? Same questions for $\sin(3x)$.

10. Suppose you are integrating $P(x)/Q(x)$, where P and Q are polynomials. Explain the process by which we integrate this expression. (Consider the degrees of P and Q).

11. What was the Mean Value Theorem for Integrals?

12. Evaluate: $\int_0^\infty t\mathrm{e}^{-st}\ dt$, where s is a positive constant.

13. Write the following limit as a definite integral: on the given interval:
$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(2+\frac{\pi i}{n}\right) \sin\left(\frac{2+\pi i}{n}\right) \cdot \frac{\pi}{n}.$$

14. If $f(x) = 1 + x + x^2 + x^3 + \ldots = \sum_{n=0}^\infty x^n$,
    (a) Find the interval on which the sum converges.
    (b) Viewing this as a geometric series, what is the formula for f?
    (c) Find $f'(x)$ by differentiating the power series. If you also differentiate your answer to part (b), then you'll get a formula for the new power series.
    (d) Find $\int f(x)\,dx$ by integrating the power series. What is the formula for the resulting sum (using the antiderivative of part (b))?

15. Find the area between the curves $y^2 = x$ and $x - 2y = 3$.

16. Write the following difference as a single integral: $\int_2^{10} f(x)\ dx - \int_2^7 f(x)\ dx$

17. If $\int_0^1 f(x)dx = 2$, $\int_0^4 f(x)dx = -6$, and $\int_3^4 f(x)\ dx = 1$, find $\int_1^3 f(x)\ dx$.

18. If $\int_0^1 f(x)\ dx = \frac{1}{3}$, what is $\int_0^1 5 - 6f(x)\ dx$?

19. Compute $\frac{dF}{dx}$, if $F(x) = \int_x^2 \cos(t^2)\ dt$

20. Compute $\frac{dg}{dy}$, if $g(y) = \int_3^{\sqrt{y}} \frac{\cos(t)}{t}\ dt$.

21. Find $\frac{dy}{dx}$, if $y = \int_{\cos(x)}^{5x} cos(t^2)\ dt$

22. If $a_n = \left(\frac{n+1}{n}\right)^n$, what is the limit of the sequence? Would the associated series converge?

23. Evaluate the limit by recognizing the sum as the Riemann sum to an associated integral:

$$\lim_{n\to\infty} \frac{1}{n}\left(\sqrt{\frac{1}{n}}+\sqrt{\frac{2}{n}}+\ldots+\sqrt{\frac{n}{n}}\right)$$

24. Write the following integral as the limit of a Riemann sum (use right endpoints): $\displaystyle\int_0^5 (1+2x^3)\, dx$

25. Given that $\int_4^9 \sqrt{x}\, dx = \frac{38}{3}$, what is $\int_9^4 \sqrt{t}\, dt$?

26. Let $f(x) = \mathrm{e}^x$ on the interval $[0, 2]$. (a) Find the average value of f. (b) Find c such that $f_{\text{avg}} = c$.

27. The velocity function is $v(t) = 3t - 5$, $0 \le t \le 3$ (a) Find the displacement, (b) Find the distance traveled.

28. Exercise 7, pg. 427 (There are some graphs to consider).

29. Suppose $h(1) = -2$, $h'(1) = 2$, $h''(1) = 3$, $h(2) = 6$, $h'(2) = 5$, and $h''(2) = 13$, and h'' is continuous. Evaluate $\int_1^2 h''(u)\, du$.

30. Find the area between the curves $y = |x|$ and $y = x^2 - 2$.

31. If f is continuous and $\int_0^9 f(x)\ dx = 4$, find $\int_0^3 xf(x^2)\, dx$.

32. For each function, find the Taylor series for $f(x)$ centered at the given value of a:

    (a) $f(x) = 1 + x + x^2$ at $a = 2$

    (b) $f(x) = \mathrm{e}^x$ at $a = 3$.

    (c) $f(x) = \frac{1}{x}$ at $a = 1$.

33. Problem 41, p. 439 (Pictures)

34. Find a so that half the area under the curve $y = \frac{1}{x^2}$ lies in the interval $[1, a]$ and half of the area lies in the interval $[a, 4]$.

35. Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by $y = x$, $y = 4x - x^2$, about $x = 7$.

36. Compute:

    (a) $\displaystyle\frac{d}{dx}\int_{3x}^{\sin(x)} t^3\, dt$

    (b) $\displaystyle\frac{d}{dx}\int_1^5 x^3\, dx$

    (c) $\displaystyle\int_1^5 \frac{d}{dx}x^3\, dx$

37. Define the integral of f:

    (a) if f is continuous on $[a, b]$ (as a Riemann Sum)

    (b) On the interval $[a, b]$ if f has a vertical asymptote at $x = a$, but is continuous on $(a, b]$.

    (c) On the interval $[a, \infty)$, if f is continuous there.

38. What is the difference between a sequence, a series, and a power series?

39. What does it mean for a infinite series to "converge"? Be specific in your answer using s_n as the sum for $k = 1$ to n.

40. What does it mean (graphically) for a sequence to "converge"?

41. If $a_n = \frac{n!}{(n+2)!}$, does the sequence converge? If so, to what does it converge?

42. If we think of a_n as $f(n)$, then what is the relationship between $\sum_{n=1}^{T} a_n$ and $\int_1^T f(x)\, dx$? You may assume f is decreasing and positive. Hint: There are two possibilities, where you use either right endpoints or left endpoints in a Riemann sum.

43. The function $f(x)$ is given as straight lines going through the points $(0, 1)$, $(2, 3)$, and $(5, 0)$. Compute $\int_0^5 f(x)\, dx$ using geometry.

Practice with Series: Converge (absolute or conditional) or Diverge?

1. $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{n^2+n}$

2. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{3n}{1+8n}\right)^{n}$

3. $\displaystyle\sum_{n=1}^{\infty}\frac{10^n}{n!}$

4. $\displaystyle\sum_{n=1}^{\infty}n^2\mathrm{e}^{-n^3}$

5. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{\sqrt{n}-1}$

6. $\displaystyle\sum_{k=1}^{\infty}\frac{k+5}{5^k}$

7. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n n}{(n+1)(n+2)}$

8. $\displaystyle\sum_{n=1}^{\infty}\frac{\sqrt{n^2-1}}{n^3+2n^2+5}$

9. $\displaystyle\sum_{n=1}^{\infty}\frac{\cos(n/2)}{n^2+4n}$

10. $\displaystyle\sum_{n=1}^{\infty}\frac{n^2+1}{5^n}$

11. $\displaystyle\sum_{k=1}^{\infty}(-1)^k\frac{\sqrt{k}}{k+5}$

12. $\displaystyle\sum_{n=1}^{\infty}\frac{(2n)^n}{n^{2n}}$

13. $\displaystyle\sum_{n=1}^{\infty}\frac{3^n n^2}{n!}$

14. $\displaystyle\sum_{k=1}^{\infty}k^{-1.7}$

Practice with Power Series: Find the interval of convergence.

1. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n x^n}{n+1}$

2. $\displaystyle\sum_{n=1}^{\infty}\frac{x^n}{n^2}$

3. $\displaystyle\sum_{n=1}^{\infty}n^n x^n$

4. $\displaystyle\sum_{n=1}^{\infty}\frac{n^2 x^n}{10^n}$

5. $\displaystyle\sum_{n=1}^{\infty}\frac{(3x-2)^n}{n3^n}$

6. $\displaystyle\sum_{n=1}^{\infty}n^3(x-5)^n$

7. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n x^{2n-1}}{(2n-1)!}$

8. $\displaystyle\sum_{n=1}^{\infty}\frac{(4x+1)^n}{n^2}$

9. Evaluate:

(a) $\displaystyle\int\frac{\sec^2(x)}{1-\tan(x)}\,dx$

(b) $\displaystyle\int_0^{\infty}\frac{1}{(x+2)(x+3)}\,dx$

(c) $\displaystyle\int\ln(x)\,dx$

(d) $\displaystyle\int\frac{dx}{x\ln(x)}$

(e) $\displaystyle\int\frac{1}{y^2-4y-12}\,dy$

(f) $\displaystyle\int(1-t)(2+t^2)\,dt$

(g) $\displaystyle\int u(\sqrt{u}+\sqrt[3]{u})\,du$

(h) $\displaystyle\int_1^{\infty}\frac{1}{(2x+1)^3}\,dx$

(i) $\displaystyle\int_0^{3}\frac{1}{\sqrt{x}}\,dx$

(j) $\displaystyle\int_1^{4}\frac{\mathrm{e}^{1/x}}{x^2}\,dx$

(k) $\int e^{-x} \sin(2x)\, dx$

(l) $\int \frac{x^2}{(4-x^2)^{3/2}}\; dx$

(m) $\int \frac{1}{1+e^x}\; dx$

(n) $\int \frac{\tan^{-1}(x)}{1+x^2}\; dx$

(o) $\int_{-a}^{a} x\sqrt{x^2-a^2}\; dx$

(p) $\int_0^2 \frac{dx}{(2x-3)^2}\; dx$

(q) $\int \sin^2 x \cos^5 x\; dx$

(r) $\int \frac{\sqrt{9x^2-4}}{x}\; dx$

(s) $\int \frac{1}{\sqrt{x^2-4x}}\; dx$

(t) $\int x^4 \ln(x)\; dx$

(u) $\int \frac{2}{3x+1} + \frac{2x+3}{x^2+9}\, dx$

(v) $\int x^2 \cos(3x)\, dx$