Chapter 5 Review Solutions

The assigned problems in the Chapter 5 Review: True/False:1-7,11-14, and in the Review: 2, 5, 7, 8, 10, 12, 15, 17, 18, 21, 22, 27, 29, 31, 32, 33, 36, 38, 40, 48, 50, 67, 68, 69, 71, 72

You should also go through the previous handout with the Riemann Sums.

True or False:

1. True, this is a property of integrals. (Section 5.2)
2. False. This is not a property of integrals. For example, try $a = 0, b = 2$, $f(x) = 1$, $g(x) = 1$.
3. True, this is a property of integrals. (Sect. 5.2)
4. False. We cannot take a variable outside of the integral sign. For example, try $f(x) = 1$, $a = 0$, $b = 1$.
5. False. For example, try $f(x) = x^2$, $a = 0, b = 1$.
6. True. This is the Fundamental Theorem of Calculus, part II.
7. True, this is a property of integrals.
11. False. There is a vertical asymptote at $x = 0$, so this integral is not defined.
12. False. The actual area is given by $\int_0^2 |x - x^3|\, dx$, since $x - x^3$ is negative for $0 \leq x \leq 1$ (use a sign chart).
13. False. For example, $f(x) = |x|$ is continuous, but not differentiable at $x = 0$.
14. True. This is the Fundamental Theorem of Calculus, part I.

Review Problems:

2. (a) We get: $0.5(-0.25 + 0 + 0.75 + 2) = 1.25$

   (b) Setting up the Riemann sum,

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2}{n}\right)\left(\frac{4i^2}{n^2} - \frac{2i}{n}\right) = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{8i^2}{n^3} - \frac{4i}{n^2}\right) =$$

$$\lim_{n\to\infty} \left(\frac{8}{n^3}\sum_{i=1}^{n} i^2 - \frac{4}{n^2}\sum_{i=1}^{n} i\right) = \lim_{n\to\infty} \left(\frac{8}{n^3}\frac{n(n+1)(2n+1)}{6} - \frac{4}{n^2}\frac{n(n+1)}{2}\right) =$$

$$\lim_{n\to\infty} \left(\frac{4}{3}\cdot\frac{n+1}{n}\cdot\frac{2n+1}{n} - 2\cdot\frac{n+1}{n}\right) = \frac{8}{3} - 2 = \frac{2}{3}$$

   (c) $\displaystyle\int_0^2 x^2 - x\, dx = \frac{1}{3}x^3 - \frac{1}{2}x^2\Big|_0^2 = \frac{8}{3} - 2 = \frac{2}{3}$

5. $\displaystyle\int_0^6 f(x)\, dx = \int_0^4 f(x)\, dx + \int_4^6 f(x)\, dx$. So, $\displaystyle\int_4^6 f(x)\, dx = 10 - 7 = 3$.

7. c is the graph of f, b is the graph of $f'(x)$, a is the graph of $\int_0^x f(t)\, dt$.

8. (a) By FTC (part 2), $\displaystyle\int_0^{\pi/2} \frac{d}{dx}\left(\sin(x/2)\cos(x/3)\right)\, dx = \left(\sin(x/2)\cos(x/3)\right|_0^{\pi/2} = \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} - 1 = \frac{\sqrt{6}}{4}$

   (b) We recognize that $\int_0^{\pi/2} \sin(t/2)\cos(t/3)\, dt$ is a constant, so its derivative is 0.

   (c) By the FTC, part 1:

$$\frac{d}{dx}\int_x^{\pi/2} \sin(t/2)\cos(t/3)\, dt = -\frac{d}{dx}\int_{\pi/2}^{x} \sin(t/2)\cos(t/3)\, dt = \sin(x/2)\cos(x/3)$$

10. Nothing tricky here- just antidifferentiate by part 2, and get: $\frac{1}{4}b^2 + 2b^2 - b$.

12. Let $u = 1 - x$, $du = -\,dx$. Furthermore, if $x = 0, u = 1$ and if $x = 1, u = 0$. Substituting these in:

$$\int_0^1 (1-x)^9\,dx = -\int_1^0 u^9\,du = \int_0^1 u^9\,du = \left(\frac{x^{10}}{10}\right|_0^1 = \frac{1}{10}$$

15. Let $u = 1 + 2x^3, du = 6x^2\,dx$. If $x = 0, u = 1$, and if $x = 2, u = 17$. Substitution gives:

$$\int_0^2 x^2(1+2x^3)^3\,dx = \frac{1}{6}\int_1^{17} u^3\,du = \left(\frac{u^4}{24}\right|_1^{17} = 3480$$

17. Let $u = 2x + 3, du = 2\,dx$ If $x = 3, u = 9$ and if $x = 11, u = 25$. Substitution yields

$$\int_3^{11} \frac{dx}{sqrt2x+3} = \frac{1}{2}\int_9^{25} u^{-1/2}\,du = \left(u^{1/2}\right|_9^{25} = 5 - 3 = 2$$

18. The integral does not exist since the integrand has a vertical asymptote at $x = 1$, which is inside the interval of integration.

21. The antiderivative of $\mathrm{e}^{\pi t} = \frac{1}{\pi}\mathrm{e}^{\pi t}$. (or use substitution with $u = \pi t, du = \pi\,dt$). The answer is $\frac{1}{\pi}(\mathrm{e}^{\pi} - 1)$

22. Let $u = 2 - 3x, du = -3\,dx$ If $x = 1, u = -1$ If $x = 2, u = -4$. Substitute:

$$\int_1^2 \frac{1}{2-3x}\,dx = \frac{-1}{3}\int_{-1}^{-4} \frac{du}{u} = \frac{-1}{3}\left(\ln|u|\right|_{-1}^{-4} = \frac{-\ln(4)}{3}$$

27. Let $u = \pi x, du = \pi\,dx$ Thus, we get $\frac{1}{\pi}\int \sin(u)\,du = -\frac{\cos(\pi x)}{\pi} + C$

29. Let $u = \frac{1}{t}$, so that $du = \frac{-1}{t^2}\,dt$. Substitution:

$$\int \frac{cos(1/t)}{t^2}\,dt = -\int \cos(u)\,du = -\sin(u) + C = -\sin(1/t) + C$$

31. Let $u = \sqrt{x}$, so $du = \frac{1}{2\sqrt{x}}\,dx$. Substitution:

$$\int \frac{\mathrm{e}^{\sqrt{x}}}{\sqrt{x}} = 2\int \mathrm{e}^u\,du = 2\mathrm{e}^u + C = 2\mathrm{e}^{\sqrt{x}} + C$$

32. Let $u = \ln(x)$, so $du = \frac{1}{x}\,dx$. Substitution:

$$\int \frac{\cos(\ln(x))}{x}\,dx = \int \cos(u)\,du = \sin(u) + C = \sin(\ln(x)) + C$$

33. Let $u = \ln(cos(x))$ so $du = \frac{-\sin(x)}{\cos(x)}\,dx = -\tan(x)\,dx$. Substitution:

$$\int \tan(x)\ln(\cos(x))\,dx = -\int u\,du = -\frac{1}{2}u^2 + C = -\frac{1}{2}(\ln(\cos(x)))^2 + C$$

36. Let $u = \ln(e^x + 1)$, so $du = \frac{e^x}{e^x+1}\,dx$ Now,

$$\int \frac{e^x}{(e^x+1)\ln(e^x+1)}\,dx = \int \frac{du}{u} = \ln|u| + C = \ln\left(\ln(e^x+1)\right) + C$$

(NOTE: The absolute value sign can go away since $e^x + 1$ is always greater than 1, so $\ln(e^x + 1)$ will always be positive.)

38. Let $u = 1 + \tan(t)$, so $du = \sec^2(t)\,dt$. If $x = 0, u = 1$ and if $x = \frac{\pi}{4}$, $u = 2$. Substitution:

$$\int_0^{\pi/4} (1 + \tan(t))^3 \sec^2(t)\,dt = \int_1^2 u^3\,du = \left(\frac{1}{4}u^4\right|_1^2 = \frac{15}{4}$$

40. Do a sign chart analysis on the integrand, $x^2 - 6x + 8 = (x - 2)(x - 4)$. Doing this, we find that $x^2 - 6x + 8 \geq 0$ if $x \leq 2$ or $x \geq 4$. So the integral gets split into three pieces:

$$\int_0^2 x^2 - 6x + 8\,dx + \int_2^4 -(x^2 - 6x + 8)\,dx + \int_4^8 x^2 - 6x + 8\,dx$$

Evaluating, we get:

$$\left[\left(\frac{8}{3} - 12 + 16\right) - 0\right] - \left[\left(\frac{64}{3} - 48 + 32\right) - \left(\frac{8}{3} - 12 + 16\right)\right] +$$

$$\left[\left(\frac{512}{3} - 192 + 64\right) - \left(\frac{64}{3} - 48 + 32\right)\right] = \frac{136}{3}$$

48. We define $g(x) = \int_1^x \sqrt[3]{1 - t^2}\,dt$. Then:

$$\int_1^{\cos(x)} \sqrt[3]{1 - t^2}\,dt = g(\cos(x))$$

so the derivative is $g'(\cos(x))(-\sin(x)) = -\sqrt[3]{1 - (\cos(x))^2}\sin(x)$. This could be simplified further, but this is sufficient.

50. Define $g(x) = \int_1^x \sin(t^4)\,dt$. Then

$$\int_{2x}^{3x+1} \sin(t^4)\,dt = g(3x + 1) - g(2x)$$

so the derivative is: $g'(3x + 1) \cdot 3 - g'(2x) \cdot 2$, which is:

$$3\sin((3x + 1)^4) - 2\sin((2x)^4)$$

67. Differentiate both sides of the equation, and solve for $f(x)$:

$$\int_0^x f(t)\,dt = x\mathrm{e}^{2x} + \int_0^x \mathrm{e}^{-t} f(t)\,dt \Rightarrow f(x) = \mathrm{e}^{2x} + 2x\mathrm{e}^{2x} + \mathrm{e}^{-x} f(x)$$

$$f(x)(1 - \mathrm{e}^{-x}) = \mathrm{e}^{2x}(1 + 2x) \Rightarrow f(x) = \frac{\mathrm{e}^{2x}(1 + 2x)}{1 - \mathrm{e}^{-x}}$$

68. From the FTC part II, $\int_1^2 h''(u)\,du = h'(2) - h'(1) = 5 - 2 = 3$. The other information is not necessary.

69. Let $u = f(x)$, so that $du = f'(x)\,dx$. If $x = a$, $u = f(a)$ and if $x = b$, $u = f(b)$. Substituting into the given integral, we have:

$$2\int_{f(a)}^{f(b)} u\,du = u^2\Big|_{f(a)}^{f(b)} = (f(b))^2 - (f(a))^2$$

71. We'll simplify the integral $\int_0^1 f(1 - x)\,dx$, and show that it is $\int_0^1 f(x)\,dx$ using substitution:
Let $u = 1 - x$. Then $du = -dx$, and if $x = 0$, $u = 1$, and if $x = 1$, $u = 0$. We have

$$\int_0^1 f(1 - x)\,dx = -\int_1^0 f(u)\,du = \int_0^1 f(u)\,du = \int_0^1 f(x)\,dx$$

72. Notice that this sum is the definite integral:

$$\int_0^1 x^9\,dx = \frac{1}{10}x^{10}\Big|_0^1 = \frac{1}{10}$$