Review Questions, Calc I and App. E, 5.1-5.2

SOLUTIONS:

1. Put together a sign chart to see where

$$\frac{1-x^2}{x^2-4} \geq 0, \text{ or } \frac{(1-x)(1+x)}{(x+2)(x-2)} \geq 0$$

$(1-x)$	$+$	$+$	$+$	$-$	$-$
$(1+x)$	$-$	$-$	$+$	$+$	$+$
$(x+2)$	$-$	$+$	$+$	$+$	$+$
$(x-2)$	$-$	$-$	$-$	$-$	$+$
	$x < -2$	$-2 < x < -1$	$-1 < x < 1$	$1 < x < 2$	$x > 2$

From this we see the solution is: $-2 < x \leq -1$, or $1 \leq x < 2$

2. (a) $\frac{1}{2}$ This was obtained by multiplying the numerator out:

$$\lim_{n\to\infty} \frac{3n^3+5n^2+2n}{6n^3+2n+1} = \lim_{n\to\infty} \frac{(3n^3+5n^2+2n)/n^3}{(6n^3+2n+1)/n^3} = \lim_{n\to\infty} \frac{3+\frac{5}{n}+\frac{2}{n^2}}{6+\frac{2}{n^2}+\frac{1}{n^3}} = \frac{3}{6} = \frac{1}{2}$$

(b) -1
This was obtained by dividing by $\sqrt{n}$:

$$\lim_{n\to\infty} \frac{1-\sqrt{n}}{1+\sqrt{n}} = \lim_{n\to\infty} \frac{(1-\sqrt{n})/\sqrt{n}}{(1+\sqrt{n})/\sqrt{n}} = \lim_{n\to\infty} \frac{\frac{1}{\sqrt{n}}-1}{\frac{1}{\sqrt{n}}+1} = -1$$

(c) $6+\frac{18}{5} = \frac{48}{5} = 9.6$ Notice that:

$$\lim_{n\to\infty} 6+\frac{18}{5n^2}\cdot n(n+2) = 6+\frac{18}{5}\lim_{n\to\infty}\frac{n^2+2n}{n^2} = 6+\frac{18}{5}\lim_{n\to\infty}\frac{(n^2+2n)/n^2}{n^2/n^2} =$$

$$6+\frac{18}{5}\lim_{n\to\infty}\left(1+\frac{2}{n}\right) = 6+\frac{18}{5}$$

(d) 0
Remember our "trick": Rationalize!

$$\lim_{x\to\infty}\left(\sqrt{x^2+1}-\sqrt{x^2-1}\right)\frac{\sqrt{x^2+1}+\sqrt{x^2-1}}{\sqrt{x^2+1}+\sqrt{x^2-1}} =$$

$$\lim_{x\to\infty}\frac{x^2+1-(x^2-1)}{\sqrt{x^2+1}+\sqrt{x^2-1}} =$$

$$\lim_{x\to\infty}\frac{2}{\sqrt{x^2+1}+\sqrt{x^2-1}} = 0$$

3. The following answers are not unique, but some possibilities are listed for each. They all depend on how you define a:

(a) $\int_0^1 1+x^3\,dx$ or $\int_1^2 1+(x-1)^3\,dx$

(b) $\int_1^4 \sqrt{x}\,dx$ or $\int_0^3 \sqrt{1+x}\,dx$

(c) $\int_0^1 \frac{1}{1+x^2}\,dx$ or $\int_1^2 \frac{1}{1+(x-1)^2}\,dx$

4. (a) $F(x) = x^2 + 5\sin^{-1}(x) + C$
   (b) $F(x) = -x^{-1} + x^{-3}$
   (c) $F(x) = 4x - 3\tan^{-1}(x) + C$. Since we want $F(1) = 0$, we find the right C:

$$F(1) = 4 - 3\tan^{-1}(1) = 4 - 3\frac{\pi}{4}$$

   so $-4 + \frac{3\pi}{4} = C$.

5. For each of the following, we use rectangles of equal width and right endpoints:

   (a) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left(1 + \left(2 + \frac{i}{n}\right)\right)$

   (b) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi}{n}\cos\left(\pi + \frac{\pi i}{n}\right)$

   (c) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{4}{n}\left[2 + \left(-1 + \frac{4i}{n}\right) + 3\left(-1 + \frac{4i}{n}\right)^2\right]$

6. (a) $\frac{1}{3}$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^2}{n^3} = \lim_{n\to\infty} \frac{1}{n^3}\sum_{i=1}^{n} i^2 = \lim_{n\to\infty} \frac{1}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} = \frac{2}{6}$$

   (b) 5/4

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i^3}{n^4} + \frac{1}{n}\right) = \lim_{n\to\infty} \frac{1}{n^4}\sum_{i=1}^{n} i^3 + 1 = \lim_{n\to\infty} \frac{1}{n^4} \cdot \frac{n^2(n+1)^2}{4} + 1 = \frac{1}{4} + 1$$

   (c) 14

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{16i^3}{n^4} + \frac{20i}{n^2}\right] = \frac{16}{n^4} \cdot \frac{n^2(n+1)^2}{4} + \frac{20}{n^2} \cdot \frac{n(n+1)}{2} = 4 \cdot \frac{(n+1)^2}{n^2} + 10 \cdot \frac{n+1}{n} = 14$$

7. As previously, the integral will depend on what you define a to be, so here are some choices:

   (a) $\int_0^1 x^2\,dx$, or $\int_1^2 (x-1)^2\,dx$

   (b) $\int_0^1 1 + x^3\,dx$, or $\int_1^2 1 + (x-1)^3\,dx$

   (c) $\int_0^2 x^3 + 5x\,dx$, or $\int_1^3 (x-1)^3 + 5(x-1)\,dx$