Final Exam Review
Calculus II
Sheet 3

1. True or False, and give a short reason:

   (a) The definition of an infinite sum involves having the terms of the sum go to zero "fast enough".

   (b) The Ratio Test will not give a conclusive result for $\sum \frac{2n+3}{3n^4+2n^3+3n+5}$

   (c) If $\displaystyle\sum_{n=k}^{\infty} a_n$ converges for some large k, then so will $\displaystyle\sum_{n=1}^{\infty} a_n$.

   (d) If f is continuous on $[0,\infty)$ and $\lim_{x\to\infty} f(x) = 0$, then $\int_0^\infty f(x)\,dx$ converges.

   (e) Any polynomial with real coefficients can be factored into a product of linear polynomials with real coefficients.

   (f) All right cylinders (the wall of the cylinder is at a right angle to the base) whose bases have the same area and whose heights are equal have the same volume.

   (g) If f is continuous and $\displaystyle\int_0^9 f(x)\,dx = 4$, then $\displaystyle\int_0^3 xf(x^2)\,dx = 4$.

2. Determine a definite integral representing: $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n}\frac{3}{n}\sqrt{1+\frac{3i}{n}}$ [For extra practice, try writing the integral so that the left endpoint (or bottom bound) must be 5].

3. Let $\displaystyle\Gamma(n) = \int_0^\infty x^{n-1}\mathrm{e}^{-x}\,dx$

   - Show that $\Gamma(1) = 1$
   - Show that $\Gamma(n+1) = n!$, if n is a positive integer (Hint: Integration by parts using a table)

   [*This is the "Gamma Function", which is how one would compute the factorial of a non-integer number. The first exercise shows why* $0! = 1$.]

4. Evaluate $\displaystyle\int_2^5 (1+2x)\,dx$ by using the definition of the integral.

5. For each function, find the Taylor series for $f(x)$ centered at the given value of a:

   (a) $f(x) = 1 + x + x^2$ at $a = 2$

   (b) $f(x) = \frac{1}{x}$ at $a = 1$.

6. Find a so that half the area under the curve $y = \frac{1}{x^2}$ lies in the interval $[1, a]$ and half of the area lies in the interval $[a, 4]$.

7. Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by $y = x$, $y = 4x - x^2$, about $x = 7$.

8. Evaluate each of the following:

   (a) $\displaystyle\frac{d}{dx}\int_{3x}^{\sin(x)} t^3\,dt$

   (b) $\displaystyle\frac{d}{dx}\int_1^5 x^3\,dx$

   (c) $\displaystyle\int_1^5 \frac{d}{dx}x^3\,dx$

Converge (absolute or conditional) or Diverge?

9. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n n}{(n+1)(n+2)}$

10. $\displaystyle\sum_{n=1}^{\infty}\frac{\sqrt{n^2-1}}{n^3+2n^2+5}$

11. $\displaystyle\sum_{k=1}^{\infty}\frac{4^k+k}{k!}$

Find the interval of convergence.

12. $\sum_{n=1}^{\infty} n^n x^n$

13. $\sum_{n=1}^{\infty} \frac{(x+2)^n}{n4^n}$

14. $\sum_{n=1}^{\infty} \frac{2^n(x-3)^n}{\sqrt{n+3}}$

Evaluate:

15. $\int_0^{\infty} \frac{1}{(x+2)(x+3)}\, dx$

16. $\int u(\sqrt{u} + \sqrt[3]{u})\, du$

17. $\int \frac{x^2}{(4-x^2)^{3/2}}\, dx$

18. $\int \frac{\tan^{-1}(x)}{1+x^2}\, dx$

19. $\int \frac{1}{\sqrt{x^2-4x}}\, dx$

20. $\int x^4 \ln(x)\, dx$