# Summary of Patterns in Integration by Parts

The General Statement: Integration by parts comes from a "Backwards Product Rule":

$$\int u\,dv = uv - \int v\,du$$

**Pattern 1:** $\int x^n f(x)\,dx$

In this case, make a table, using $u = x^n$, $dv = f(x)\,dx$. Differentiating the $u$ column will give you zero at some point. Works when $f(x)$ is easily antidifferentiated (like $e^{kx}$, $\sin(kx)$, $\cos(kx)$).

**Example:** $\int x^3 e^{2x}\,dx$

| Sign | $u$ | $dv$ |
|---|---|---|
| + | $x^3$ | $e^{2x}$ |
| − | $3x^2$ | $(1/2)e^{2x}$ |
| + | $6x$ | $(1/4)e^{2x}$ |
| − | $6$ | $(1/8)e^{2x}$ |
| + | $0$ | $(1/16)e^{2x}$ |

$$\Rightarrow \int x^3 e^{2x}\,dx = \left(\frac{x^3}{2} - \frac{3x^2}{4} + \frac{6x}{8} - \frac{6}{16}\right)e^{2x} + C$$

**Pattern 2:** $\int \ln(x)\,dx$, $\int \sin^{-1}(x)\,dx$, $\int \tan^{-1}(x)\,dx$

The idea for this one comes from the fact that the derivatives of $\ln(x)$, $\sin^{-1}(x)$, $\tan^{-1}(x)$ are all algebraic expressions of $x$. The general technique is to let $u = f(x)$, and $dv = dx$, so that $\int f(x)\,dx = xf(x) - \int xf'(x)\,dx$

**Example:** $\int \ln(x)\,dx$

| Sign | $u$ | $dv$ |
|---|---|---|
| + | $\ln(x)$ | $1$ |
| − | $(1/x)$ | $x$ |

$$\Rightarrow \int \ln(x)\,dx = x\ln(x) - \int x\frac{1}{x}\,dx = x\ln(x) - x + C$$

**Pattern 3:** $\int e^{kx}\sin(nx)\,dx$, $\int e^{kx}\cos(mx)\,dx$

The idea for this one comes from the fact that the derivatives of $e^{kx}$ give back a constant times $e^{kx}$, coupled with the second derivative/antiderivative of $\sin(nx)$ or $\cos(mx)$ giving back a constant times $\sin(nx)$ or $\cos(mx)$ (respectively). So, use integration by parts twice (using a table) to get the same integral on both sides of the equation, then solve for the integral.

**Example:** $\int e^{2x}\cos(3x)\,dx$

| Sign | $u$ | $dv$ |
|---|---|---|
| + | $e^{2x}$ | $\cos(3x)$ |
| − | $2e^{2x}$ | $(1/3)\sin(3x)$ |
| + | $4e^{2x}$ | $-(1/9)\cos(3x)$ |

$$\Rightarrow \int e^{2x}\cos(3x)\,dx = \frac{1}{3}e^{2x}\sin(3x) + \frac{2}{9}e^{2x}\cos(3x) - \frac{4}{9}\int e^{2x}\cos(3x)\,dx$$

Now add $\frac{4}{9}\int e^{2x}\cos(3x)\,dx$ to both sides:

$$\frac{13}{9}\int e^{2x}\cos(3x)\,dx = \frac{1}{3}e^{2x}\sin(3x) + \frac{2}{9}e^{2x}\cos(3x)$$

To get a final answer:

$$\int e^{2x}\cos(3x)\,dx = \frac{3}{13}e^{2x}\sin(3x) + \frac{2}{13}e^{2x}\cos(3x) + C$$