Review Questions, 5.3-5.5

Plus a few others!

You should also look through your homework questions and previous quizzes. For more problems like the ones on this sheet, see the Chapter 5 Review, p 426

1. True or False, and give a short reason:
   (a) If f and g are continuous on $[a,b]$, then $\int_a^b f(x)+g(x)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$
   (b) If f and g are continuous on $[a,b]$, then $\int_a^b f(x)g(x)\,dx = \int_a^b f(x)\,dx \cdot \int_a^b g(x)\,dx$
   (c) If f is continuous on $[a,b]$, then $\int_a^b xf(x)\,dx = x\int_a^b f(x)\,dx$
   (d) If f' is continuous on $[-1,4]$, then $\int_{-1}^4 f'(w)\,dw = f(4) - f(-1)$
   (e) $\int_{-2}^1 \frac{1}{x^4}\,dx = -\frac{3}{8}$
   (f) All continuous functions have derivatives.
   (g) All continuous functions have antiderivatives.
   (h) If $v(t)$ is velocity at time t, then the distance traveled between times 3 and 7 is given by $\int_3^7 v(t)\,dt$
   (i) Even though the function:
   $$f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ 3+x & \text{if } x > 1 \end{cases}$$
   is not continuous at $x = 1$, we can compute $\int_0^2 f(x)\,dx$.
2. Compare the notation:
   (a) $\frac{d}{dx}\int_a^x f(t)\,dt$
   (b) $\frac{d}{dx}\int_a^b f(t)\,dt$
   (c) $\int_a^b \frac{d}{dx} f(x)\,dx$
   (d) $\int_a^b f(x)\,dt$
3. Evaluate, where possible. If not, state why:
   (a) $\int_1^4 \frac{x^2 - x + 1}{\sqrt{x}}\,dx$
   (b) $\int_0^2 \frac{x}{(x^2-1)^2}\,dx$
   (c) $\frac{d}{dx}\int_3^{3^x} t\,dt$
   (d) $\int (1-x)\sqrt{2x-x^2}\,dx$
   (e) $\int \frac{\cos(\ln(x))}{x}\,dx$
   (f) $\int_0^1 \frac{d}{dx}\left(\frac{e^x}{x+1}\right)dx$
   (g) $\int_0^{2\pi} |\sin(x)|\,dx$
   (h) $\int \frac{x}{\sqrt{1-x^4}}\,dx$
   (i) $\frac{d}{dx}\int_{2x}^{3x+1} \sin(t^4)\,dt$
   (j) $\int \frac{x^2}{\sqrt{1-x}}\,dx$
4. If f is continuous and $\int_0^4 f(x)\,dx = 10$, find $\int_0^2 f(2x)\,dx$
5. If $g(x) = \int_0^x \frac{1}{1+t+t^2}\,dt$, find where g is concave up.
6. If $\int_0^6 f(x)\,dx = 10$ and $\int_0^4 f(x)\,dx = 7$, find $\int_4^6 f(x)\,dx$.

Challenge Problems!

If you breezed through the previous questions, and are looking for something more challenging, try these!

1. $$\frac{d^2}{dx^2}\int_0^x \left(\int_1^{\sin(t)} \sqrt{1+u^4}\,du\right) dt$$
2. If f is a differentiable function so that: $\int_0^x f(t)\,dt = (f(x))^2$ for all x, find f.
3. Find
   $$\lim_{h\to 0}\frac{1}{h}\int_2^{2+h} \sqrt{1+t^3}\,dt$$