Exam 3 Notes

The third exam will cover sections 7.8, and 11.1-11.10. While 7.8 was very distinct from Chapter 11, we got some intuition about what it meant to "add an infinite number of things together" from the improper integral. Here are the main points from this material:

1. (7.8) Know how to compute an improper integral (including taking the limit). Understand the comparison test (for integrals) and apply it to determine if an integral converges or diverges.

2. Know these definitions: A sequence, series, absolute (and conditional) convergence, radius of convergence, interval of convergence, Taylor series, Maclaurin series.

3. Be able to determine if a sequence converges or diverges. This means that we can find the limit. The main techniques we reviewed were:

   (a) L'Hospital's Rule (and the algebra needed to get into a form suitable for l'Hospital)

   (b) Divide by a power of n

   As for the limit laws (and the Squeeze Theorem), you should know when you can apply them, but I won't ask you to state them.

   NOTE: You can use your intuition so that you know what you're trying to get, but for full credit, you must back up your answer with a valid technique, as one of the two listed above.

4. Understand and be able to explain what it means for a series to converge (Hint: Think about partial sums).

5. Template Series: The geometric series (and the sum of the geometric series), the $p-$series, and in particular the harmonic series and the alternating harmonic series.

   Know how we derived the sum of a geometric series.

6. Template Maclaurin Series: e^x, $\sin(x)$ and $\cos(x)$, $\frac{1}{1-x}$.

   Notice the last one is the geometric series.

7. Given $f(n) = a_n$ is positive, decreasing and continuous, graphically show why the integral test works. Know the relationship between the integral and the remainder (p. 718)

8. Be able to determine if a series converges or diverges:

   (a) Test for Divergence.

   (b) The integral test (for certain functions).

   (c) (For positive series or abs conv) The (direct or limit) comparison test.

   (d) (For abs convergence) The Ratio Test

(e) (For abs convergence) The Root test (not a common test)

(f) The Alternating Series Test and the remainder estimate for an alternating series, R_n:

$$|R_n| \leq b_{n+1}$$

NOTE: You can use your intuition so that you know what you're trying to get, but for full credit, you must back up your answer with a valid technique (like those listed above).

9. Find the radius and interval of convergence for a given power series. Understand the three possible outcomes for the radius.

10. Construct a power series for a given function using a geometric series as a template (like in 11.9). Be able to differentiate and integrate a power series.

11. Construct the Taylor (or Maclaurin) series for a given function using the formula. Recognize a given series as a special case of a template series (like 63-70 in 11.10, but I won't use the Binomial Theorem).

12. For this exam, you won't need to know the the statement of the binomial theorem.