Final Exam Review
Calculus II
Sheet 1

1. State the definition of $\int_a^b f(x)\,dx$, assuming equal subintervals and right endpoints.

2. True or False, and give a short reason:

   (a) The Alternating Series Test is sufficient to show that a series is conditionally convergent.

   (b) You can use the Integral Test to show that a series is absolutely convergent.

   (c) Consider $\sum a_n$. If $\lim_{n\to\infty} a_n = 0$, then the sum is said to converge.

   (d) The sequence $a_n = 0.1^n$ converges to $\frac{1}{1-0.1}$.

   (e) All continuous functions have derivatives.

   (f) All continuous functions have antiderivatives.

3. Set up an integral for the volume of the solid obtained by rotating the region defined by $y = \sqrt{x-1}$, $y = 0$ and $x = 5$ about the $y-$axis. Find the work involved if this was a tank filled with water and we wanted to pump it all out of the top (You may assume the measurements are in meters, and $g = 9.8$. The density of water is 1000 kg/m^3).

4. Write the area under $y = \sqrt[3]{1+x}$, $1 \le x \le 4$ as the limit of a Riemann sum (use **right** endpoints). For the same function, write an integral representing the arc length (do not evaluate the integral).

5. Find the Taylor series for $f(x) = \sqrt{x}$ centered at $a = 9$.

6. Find $\frac{dy}{dx}$, if $y = \int_{\cos(x)}^{5x} cos(t^2)\,dt$

7. Let $f(x) = \mathrm{e}^x$ on the interval $[0, 2]$. (a) Find the average value of f. (b) Find c such that $f_{\text{avg}} = f(c)$.

8. Use a template series to find the series for $\int \cos(x^2)\,dx$.

9. Does the series converge (absolute or conditional), or diverge?

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n/2)}{n^2+4n}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{5^n}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$

10. Find the interval of convergence:

    (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 x^n}{10^n}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n3^n}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n-1}}{(2n-1)!}$

11. Evaluate the integral.

(a) $\displaystyle\int \frac{1}{y^2-4y-12}\,dy$

(b) $\displaystyle\int \frac{2}{3x+1}+\frac{2x+3}{x^2+9}\,dx$

(c) $\displaystyle\int x^2\cos(3x)\,dx$

(d) $\displaystyle\int_{-2}^{2}|x-1|\,dx$

(e) $\displaystyle\int \frac{dx}{x\ln(x)}$

(f) $\displaystyle\int x\sqrt{x-1}\,dx$

(g) $\displaystyle\int_0^\infty \frac{x^2}{\sqrt{1+x^3}}\,dx$

(h) $\displaystyle\int \sin^2(x)\cos^3(x)\,dx$

(i) $\displaystyle\int \sin^{-1}(x)\,dx$

(j) $\displaystyle\int \frac{dx}{\sqrt{x^2-9}}$

12. The velocity function is $v(t) = 3t-5$, $0 \le t \le 3$ (a) Find the displacement. (b) Find the distance traveled.

13. Set up the integral that will give the length of the graph of $y = \sqrt{x-x^2}+\sin^{-1}(\sqrt{x})$, with $0 \le x \le 1$.

14. (You can use a calculator at home for the following problem, which would be suitably changed for an exam with no calculators) Find the sum of the series $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^3}$ correct to three decimal places.

15. Evaluate each sum. Hint on part (c): Partial Fractions

(a) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n\pi^{2n}}{6^{2n}(2n)!}$

(b) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}\frac{3^{n+2}}{2^{2n}}$

(c) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n(n+3)}$

16. Determine whether the integral is convergent or divergent: