Final Exam Review
Calculus II
Sheet 3

1. Determine if the series converges (absolute or conditional) or diverges:

   (a) $\sum_{n=1}^{\infty} \frac{(-1)^n \ln(n)}{n}$

   (b) $\sum_{n=1}^{\infty} \frac{e^n}{n!}$

   (c) $\sum_{n=1}^{\infty} \frac{n^3}{e^{n^4}}$

   (d) $\sum_{n=1}^{\infty} 4^{1-2n}$

2. Let $a_n = \frac{n + \ln(n)}{n^2}$.

   (a) Does the sequence $\{a_n\}$ converge or diverge? If it converges, find what it converges to.

   (b) Does the series $\sum_{n=1}^{\infty} a_n$ converge or diverge?

3. A bug is crawling along the graph of the curve $y = 3x + 1$ for x in the interval $[0, t]$. Find the distance the bug has traveled as a function of t.

4. Find the interval of convergence for each of the series:

   (a) $\sum_{n=0}^{\infty} \frac{(2x-3)^n}{n \ln(n)}$

   (b) $\sum_{n=0}^{\infty} \frac{x^n}{n+1}$

   (c) $\sum_{n=0}^{\infty} \frac{3^n x^n}{5^n}$

5. Expand the function $f(x) = \frac{2}{4-3x}$ as a power series centered at $x = 0$, and determine the values of x for which the series converges.

6. Evaluate the integral:

   (a) $\int \frac{x^2}{\sqrt{16 - x^2}} \, dx$

   (b) $\int x \sec(x) \tan(x) \, dx$

   (c) $\int x^2 e^{-2x} \, dx$

   (d) $\int \tan^{-1}(x) \, dx$

   (e) $\int \frac{x^2 - x + 1}{x^2 + x} \, dx$

   (f) $\int \frac{dx}{x^2 + 4x - 5}$

   (g) $\int_0^3 |x^2 - 4| \, dx$

   (h) $\int_1^9 \frac{\sqrt{x} - 2x^2}{x} \, dx$

   (i) $\int_{-3}^{3} \frac{\sin(x)}{x^2 + 1} \, dx$

7. Evaluate $\int \frac{dx}{x^2 - 1} dx$ two ways- Using partial fractions and using trig substitution.

8. Determine if the integral converges or diverges. If it converges, determine what it converges to. $\int_{-\infty}^{9} e^{4x} \, dx$

9. Does the integral converge or diverge (and give a short reason): $\int_8^{\infty} \sin^2(x) e^{-x} \, dx$

10. Consider the region in the first quadrant bounded by the curve $y = 9 - x^2$ with $0 \leq x \leq 3$. Consider the solid obtained by rotating that region about the x axis. Set up two integrals that represent the volume of this solid- One using shells, and one using disks.

11. Same region as before. Set up an integral representing the volume (using any appropriate technique) if the region is revolving about $x = 4$, and then if the region is revolving about $y = -2$.

12. A container weighing 50 lbs is filled with 20 ft^3 of water. The container is raised vertically at a constant speed of 2 ft/sec for 1 minute, during which time the water leaks out at a rate of 1/3 ft^3/sec. Calculate the total work performed in raising the container (ignore the rope).

13. Use the *definition* of the definite integral (with right endpoints) to calculate the value of $\int_0^2 (x^2 - x)\, dx$.

14. Find the derivative (with respect to x) of the function : $F(x) = \int_{\sqrt{x}}^{x} \frac{e^t}{t}\, dt$

15. Find the c guaranteed by the Mean Value Theorem for Integrals, if $f(x) = 1/x$ on the interval $[1, 3]$. Hint: It has something to do with the average value of f.

16. Find an integral representing the surface area for the surface obtained by rotating the graph of $y = \sin(x)$, $0 \leq x \leq \pi$ about the line $y = -1$.

17. (a) Find the Taylor series for f centered at 4 if

$$f^{(n)}(4) = \frac{(-1)^n n!}{3^n (n+1)}$$

(b) What is the radius of convergence of the Taylor series? (c) If we want to approximate f by a cubic polynomial (centered at 4), write the polynomial we would use.