GROUP WORK 2, SECTION 7.1
Find the Error

It is a beautiful Spring day. You leave your calculus class feeling sad and depressed. You aren't sad because of the class itself. On the contrary, you have just learned an amazing integration technique: Integration by Parts. You aren't sad because it is your birthday. On the contrary, you are still young enough to actually be happy about it. You are sad because you know that every time you learn something really wonderful in calculus, a wild-eyed stranger runs up to you and shows you a "proof" that it is false. Sure enough, as you cross the street, he is waiting on the other side.

"Good morning, Kiddo," he says.

"I just learned integration by parts. Let me have it."

"What do you mean?" he asks.

"Aren't you going to run around telling me that all of math is lies?"

"Well, if you insist," he chuckles... and hands you a piece of paper:

$$\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx$$

$$u = \frac{1}{\cos x} \qquad dv = \sin x\,dx$$

$$du = \tan x \sec x\,dx \qquad v = -\cos x$$

$$\int \tan x\,dx = uv - \int v\,du$$

$$\int \tan x\,dx = -1 + \int \tan x\,dx$$

$$0 = -1$$

"Hey," you say, "I don't get it! You did everything right this time!"

"Yup!" says the hungry looking stranger.

"But... Zero isn't equal to negative one!"

"Nope!" he says.

You didn't think he could pique your interest again, but he has. Spite him. Find the error in his reasoning.

GROUP WORK 2, SECTION 7.1
Find the Error (The Sequel)

What a wonderful day! You have survived another encounter with the wild-eyed stranger, demolishing his mischievous pseudo-proof. As you leave his side, you can't resist a taunt.

"Didn't your mother tell you never to forget your constants?" It seemed a better taunt when you were thinking it than it did when you said it.

"Eh?" he says. You come up to him again.

"I was just teasing you. Just pointing out that when doing indefinite integration, those constants should not be forgotten. A simple, silly error, not worthy of you." You look smug. You are the victor.

"Yup. Indefinite integrals always have those pesky constants." For some reason he isn't looking defeated. He is looking crafty.

"Right. Well, I'm going to be going now..."

"Of course, Kiddo, *definite* integrals don't have constants, sure as elephants don't have exoskeletons."

"Yes. Well, I really must be going."

Surprisingly quickly, he snatches the paper out of your hand, and adds to it. This is what it now looks like.

$$\int_{\pi/6}^{\pi/4} \tan x\,dx = \int_{\pi/6}^{\pi/4} \frac{\sin x}{\cos x}\,dx$$

$$u = \frac{1}{\cos x} \qquad dv = \sin x\,dx$$

$$du = \tan x \sec x\,dx \qquad v = -\cos x$$

$$\int_{\pi/6}^{\pi/4} \tan x\,dx = uv - \int_{\pi/6}^{\pi/4} v\,du$$

$$\int_{\pi/6}^{\pi/4} \tan x\,dx = -1 + \int_{\pi/6}^{\pi/4} \tan x\,dx$$

$$0 = -1$$

"No constants missing here! Happy Birthday!" The stranger leaves, singing the "Happy Birthday" song in a minor key. Now there are no constants involved in the argument. But the conclusion is the same: $0 = -1$. Is the stranger right? Has he finally demonstrated that all that you've learned is suspect and contradictory? Or can you, using your best mathematical might, find the error in this new version of his argument?