Selected Answers: 11.5

1, 3, 6, 12, 15, 17, 21, 24, 29, 32, 35

6. The series is alternating with $b_n = \frac{1}{\ln(n+4)} > 0$ for all $n \geq 1$. The terms are decreasing: Since $\ln(x)$ is increasing, $\ln(n+5) > \ln(n+4)$, so that

$$b_{n+1} = \frac{1}{\ln(n+5)} \leq \frac{1}{\ln(n+4)} = b_n$$

and the limit is zero. Therefore, the three conditions are satisfied and the given series converges.

12. We see that $e^{1/n} > 0$, so $b_n = e^{1/n}/n > 0$ for $n \geq 1$.

Secondly, we check that the terms are decreasing:

$$f(x) = \frac{1}{x}e^{1/x} = \frac{-1}{x^2}e^{1/x} + \frac{1}{x}\frac{-1}{x^2}e^{1/x} = e^{1/x}\left(\frac{-(x+1)}{x^2}\right)$$

This quantity is negative for all $x > -1$, so our terms are decreasing.

15. Hint: $\cos(n\pi) = (-1)^n$, so then re-write the sum.

17. Note that $\sin(x) \geq 0$ and is increasing if $0 \leq x \leq \frac{\pi}{2}$. So, if $n \geq 2$, $\sin(\pi/n) > 0$. We can differentiate:

$$f(x) = \sin\left(\frac{\pi}{n}\right) \Rightarrow f'(x) = \cos(\pi/n) \cdot \frac{-\pi}{n^2}$$

The cosine function is positive if $0 \leq x \leq \pi/2$, so if $n \geq 2$, that term of the derivative will be positive. Therefore, the overall derivative is negative.

21. You don't need to graph the values, but do notice that they seem to be bouncing around a limiting value (recall that the even partial sums are increasing to the limit, and the odd partial sums are decreasing to the limit). The first few sums are given below, with the bound on the remainder R_{10}:

n	s_n
1	1.0000
2	0.6464
3	0.8389
4	0.7139
5	0.8033
6	0.7353
7	0.7893
8	0.7451
9	0.7821
10	0.7505

$$|R_{10}| \leq b_{11} = \frac{1}{11^{3/2}} \approx 0.0275$$

24. Check that the alternating series test is satisfied:

- The series is alternating

- The terms are decreasing: Since $(n+1)5^{n+1} > n5^n$, then

$$\frac{1}{(n+1)5^{n+1}} < \frac{1}{n5^n}$$

- The limit is zero (the numerator is fixed and the denominator goes to infinity).

Note that (space included for readability):

$$b_4 = \frac{1}{45^4} = 0.0004 \qquad b_5 = \frac{1}{55^5} \approx 0.000\ \ 064$$

The 5th term is less than the desired error, so we would sum using terms up to and including b_4 (in this case, use 4 terms in the sum).

29. In this case, $b_7 \approx 0.000\ \ 004\ \ 9$ which is less than our desired error. Summing on the calculator, $s_6 \approx 0.067614$.

32. If $p > 0$, then $(n+1)^p > n^p$ so that $1/(n+1)^p < 1/n^p$, so the series is decreasing, and the limit is zero so the series converges by the Alternating Series test.

However, if $p \leq 0$, then the limit of b_n does not exist (it goes to infinity), and therefore, the series would diverge by the Test for Divergence.

Therefore, the alternating $p-$series converges iff $p > 0$.

35. You might first write out what the suggested series actually is:

$$\sum(-1)^{n-1}b_n = 1 - \frac{1}{2^2} + \frac{1}{3} - \frac{1}{4^2} + \frac{1}{5} - \frac{1}{6^2} + \cdots$$

The odd terms form a harmonic series and the even terms form a geometric series- That gives me an idea...

Suppose that the original series converged. Then the sum of this series with another convergent series would have to converge (See p. 693, Theorem 8). However, suppose I add the series given by the series $\sum 1/(2n)^2$ (which converges) below:

$$\begin{array}{l} 1 - \frac{1}{2^2} + \frac{1}{3} - \frac{1}{4^2} + \frac{1}{5} - \frac{1}{6^2} + \cdots \\ 0 + \frac{1}{2^2} + 0 + \frac{1}{4^2} + 0 + \frac{1}{6^2} + \cdots \\ \hline 1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} \cdots \end{array}$$

This sum is actually $\sum \frac{1}{2n-1}$, which diverges. Therefore, the original series could not converge.

Note: This is called a “proof by contradiction”, where we assume that our outcome is false (assume that the series converges), and show that this leads us to a false conclusion (the sum of two convergent series would diverge). This means that the original statement must have been true.