Selected Answers: 11.6

NOTE: Look at 29, delete #38, and add #10, 20, 22, 31, 35, 37.

2. Rewrite as $\sum \frac{(-1)^n 2^n}{n^2}$, then use the Ratio Test to show the series diverges (the limit is 2).

6. Use the Ratio test, and the limit is 0, so the series converges (absolutely).

10. (Added) You should find that the series converges only conditionally. To show that it does not converge absolutely, compare with $\sum 1/\sqrt{n}$.

11. Probably easiest to use a comparison test: Taking the absolute value of the terms, we have:

$$\frac{e^{1/n}}{n^3} \leq e^1 \cdot \frac{1}{n^3}$$

So absolutely convergent.

12. Comparison test may be easiest again:

$$\frac{|\sin(4n)|}{4^n} \leq \frac{1}{4^n}$$

so absolutely convergent.

15. First, recall that $\arctan(x)$ is an increasing function, and

$$\lim_{x\to\infty} \arctan(x) = \frac{\pi}{2}$$

so that

$$\frac{|\arctan(n)|}{n^2} \leq \frac{\pi}{2} \cdot \frac{1}{n^2}$$

and $\sum 1/n^2$ is a convergent p-series. Therefore, the sum converges absolutely.

17. Converges, but not absolutely- That is, the given series converges by the Alternating Series Test, but if we take the absolute value of the terms, we could use a version of the limit comparison test with $b_n = 1/n$:

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{1}{\ln(n)}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{\ln(n)} = \lim_{n\to\infty} n$$

(the last step is a simplification after using L'Hospital's rule). Since the limit is infinite, we can conclude that $1/n$ is going to zero faster than $1/ln(n)$ (intuitively, this is like having a the numerator constant and the denominator going to zero, which makes the overall fraction diverge), and this as a series diverges. Therefore, the series $\sum 1/\ln(n)$ will diverge.

More formally, we could note that $\ln(n) < n$ so that by direct comparison,

$$\frac{1}{\ln(n)} \geq \frac{1}{n}$$

and the comparison test says that our series $\sum 1/\ln(n)$ diverges.

20. (Added) The Root Test can be used, and the limit is 0.

22. (Added) The Root Test can be used, and the limit is 32 (so diverges).

25. Ratio Test: A little messy at first, but simplifies a lot.

29. This actually simplifies to $\sum 2^n$, which diverges.

31. Note that
$$\frac{a_{n+1}}{a_n} = \frac{5n+1}{4n+3}$$
so the limit (in the Ratio Test) is 5/4 (so the series diverges).

35. (Added) You should find that the first and last are inconclusive.

37. (Added) This is a little preview of 11.7.

38. (Changed- This was deleted, and #35, 37 were added.)