Section 6.4 HW Notes

5, 7, 8, 15, 17, 21, 24

For the odd problems, I'll just give the set ups.

5. The work will be the integral, or in this case, the area under the force curve. Use geometry to get the area (triangle + rectangle)

7. The spring problem requires "Hooke's Law", which says that a spring stretched x units past its natural length will have a restorative force of kx. In Exercise 7, the spring is stretched 4 inches, or 1/3 feet (best to stay with feet and lbs). The force was 10 lbs, or
$$k\frac{1}{3} = 10 \quad \Rightarrow \quad k = 30$$
Therefore, for this spring, we have a restorative force of $F(x) = 30x$ (where x is the number of feet stretched past the natural length of the spring).

   If we divide the interval from $x = 0$ to $x = 1/2$ into n equal pieces, the work done on the i^{the} subinterval over Δx feet is approximately:
$$W_i = F \cdot d \approx 30x_i^* \Delta x \quad \Rightarrow \quad W = \int_0^{1/2} 30x\, dx$$

8. Sorry about the units. Converting these, we have 25 N to keep it stretched 10 cm past natural length, or 0.1 m. That means, using Hooke's Law
$$k(0.1) = 25 \quad \Rightarrow \quad k = 250$$
And, using this, we stretch the string 5 cm or 0.05 m past natural length:
$$W = \int_0^{0.05} 250x\, dx = \cdots approx 0.31J$$

15. A cable that weighs 2 lb/ft is used to lift 800 lbs of coal up a mine shaft 500 ft deep. Find the work done.

    SOLUTION: The work can be split up and added- That is, find the work hauling up the rope, and then add that to the work hauling up just the coal.

    - The work hauling up the rope: Let x be the number of feet from the top of the shaft. Subdividing the interval $0 \le x \le 500$ into n equal subintervals, hauling up the rope at the i^{th} subinterval and x units down, we'll have $500 - x$ feet of rope (at 2 lb/ft) lifted Δx feet. The amount of work done here is then $2(500 - x)\Delta x$. Integrating, we get
$$W_{\text{rope}} = \int_0^{500} 2(500 - x)\, dx = 250,000 \text{ ft-lbs}$$

- The work hauling up the coal is easier to compute- The force doesn't change, so

$$W = F \cdot d = 800 \cdot 500 = 400,000 \text{ ft-lbs}$$

Altogether, the work is $650,000$ ft-lbs.

21. A rectangular slice of water Δx meters thick and lying x meters above the bottom has width x and volume $8x\Delta x$ cubic meters. It weighs about $9800 \cdot 8x\, Deltax$ N and must be lifted $5 - x$ meters up. The total work is then:

$$W = \int_0^3 9800 \cdot 8x \cdot (5 - x)\, dx$$

24. We use similar triangles in this problem to get the volume of water x units up from the bottom. If the large triangle has side lengths $6, 12$, then the smaller triangle of water going up x units must go across $2x$ units.

    At x units from the bottom, a slice of water has volume $10 \cdot 2x \cdot \Delta x$ cubic feet, so to get the weight, multiply by the density of water (it would be given as 62.5 lbs per ft^3). That slice must be lifted $6 - x$ feet, so the work overall is

$$\int_0^6 (62.5)(20x)(6 - x)\, dx = 1250 \int_0^6 (6x - x^2)\, dx = \cdots = 45,000 \text{ ft-lbs}$$