Section 8.1 HW Notes

3, 5, 7, 9, 13, 15

3. Straightforward. The arc length integral you should get is

$$L = \int_0^{\pi} \sqrt{1+\cos^2(x)}\, dx$$

We wanted to just set this one up, so that's as far as needed.

5. Same idea as (3)- Just set it up.

7. For this one, compute the arc length in full. First, compute and simplify the integrand:

$$f(x) = 1 + 6x^{3/2} \qquad \Rightarrow \qquad 1 + (f'(x))^2 = 1 + 81x$$

Therefore, the integral is straightforward (use $u = 1 + 81x$):

$$\int_0^1 \sqrt{1+81x}\, dx = \frac{2}{243}(1+81x)^{3/2}\Big|_0^1 = \frac{2(82^{3/2}-1)}{243}$$

9. We "plug-n-chug"- First, find and simplify the integrand, then integrate.

$$f(x) = \frac{1}{3}x^3 + \frac{1}{4}x^{-1} \qquad \Rightarrow \qquad 1 + (f'(x))^2 = x^4 + \frac{1}{2} + \frac{1}{16x^4} = \left(x^2 + \frac{1}{4x^2}\right)^2$$

Now when we take the square root, this will be easy to integrate:

$$\int_1^2 x^2 + \frac{1}{4x^2}\, dx = \frac{1}{3}x^3 - \frac{1}{4x}\Big|_1^2 = \frac{59}{24}$$

13. Same idea- Find and simplify the integrand first.

$$y = \ln(\sec(x)) \qquad \Rightarrow \qquad 1 + (y')^2 = 1 + \tan^2(x) = \sec^2(x)$$

So the arc length formula becomes:

$$\int_0^{\pi/4} \sec(x)\, dx = \ln|\sec(x) + \tan(x)||_0^{\pi/4} = \ln(\sqrt{2}+1) - \ln(1) = \ln(\sqrt{2}+1)$$

Remember that $\int \sec(x)\, dx$ will be on your table.

15. The integrand is set up in such a way as to get a perfect square (the algebra is almost identical to #13).

$$1 + (y')^2 = \left(\frac{1}{2}x + \frac{1}{2x}\right)^2$$