Selected Solutions, Section 4.9

10. Note that e^2 is a constant, so the antiderivative is $e^2 C$

11. Be sure to write $f(x) = 3x^{1/2} - 2x^{1/3}$ first!

17. The antiderivative is $-2\cos(\theta) - \tan(\theta) + C$, but notice that the C can change because $\sec(\theta)$ has a lot of vertical asymptotes, breaking up the real line. Therefore, you might specify the interval, like "for x in $(-\pi/2, \pi/2)$, and the constant can change for other intervals". Or, you can specify the intervals like they do in the text solution.

42. The hint on these types of problems is to rewrite them first. In this case,

$$f''(t) = 3t^{-1/2} \quad \Rightarrow \quad f'(t) = 3 \cdot 2t^{1/2} + C = 6t^{1/2} + C$$

Using $f'(4) = 7$, we get $C = -5$, and then

$$f(t) = 6 \cdot \frac{2}{3} t^{3/2} - 5t + C_2 = 4t^{3/2} - 5t + C_2 \qquad f(4) = 20$$

We recognize that $4^{3/2} = 2^3 = 8$, so substituting $x = 4$, we get

$$12 + C_2 = 20 \quad \Rightarrow \quad C_2 = 8$$

so $f(t) = 4t^{3/2} - 5t + 8$.

46. We should find that

$$f(t) = 2e^t - 3\sin(t) + \frac{2 - 2e^{\pi}}{\pi} t - 2$$

50. From what is given, we know that the antiderivative is $f(x) = \frac{1}{4}x^4 + C$ for some C. We also know that $y = -x$ is the equation of the tangent line at some value of x. Since $f'(x) = x^3$, that point must be at $x = -1$. From the equation of the line, $y = -(-1) = 1$, so the point $(-1, 1)$ must be on the graph of f, and so

$$f(x) = \frac{1}{4}x^4 + \frac{3}{4}$$

51. Does the derivative of f appear? It looks like graph a. Can f be the derivative of another graph? Since $f(x) = 0$ in two places, the antiderivative would have a local slope of zero there (local max, local min, or plateau)- That could be b or c- But f goes from negative to positive, so the antiderivative would be decreasing, then increasing, so the antiderivative is graph b.

52. You should find that a is the antiderivative.

53-55. We'll do one or two of these in class.

59. Given the velocity, find the displacement (or position):

$$v(t) = \sin(t) - \cos(t)$$

$$s(t) = -\cos(t) + \sin(t) + C$$

To find C, use the fact that is given- We want $s(0) = 0$:

$$-1 + 0 + C = 0 \quad \Rightarrow \quad C = 1$$

Therefore, $s(t) = -\sin(t) + \cos(t) + 1$

69. A stone was dropped off a cliff and hit the ground with a speed of 120 ft/sec. What is the height of the cliff?

SOLUTION: We assume (as usual) that there is negligible air resistance, and acceleration is due only to gravity. Therefore, our model equation is given by:

$$a(t) = -32 \qquad v(t) = -32t + C_1 \qquad s(t) = -16t^2 + C_1 t + C_2$$

where C_1 is the initial velocity (in this case, $C_1 = 0$) and C_2 is the initial position (which is our unknown).

Alternative Solution 1:

Given $s(t) = -16t^2 + C_2$, the (positive) time at which the height is zero should be the same as the time at which the velocity is -120:

$$-120 = -32t \quad \Rightarrow \quad t = \frac{15}{4} = 3.75$$

The height should be zero at this time. We can solve for the height using that:

$$0 = -16\frac{15^2}{4^2} + C_2 \quad \Rightarrow \quad C_2 = 15^2 = 225 \text{ ft}$$

Alternative Solution 2:

Given $s(t) = -16t^2 + C_2$, we know that the ball hit the ground when $s(t)$=0, or

$$t = \sqrt{C_2 4}$$

We want to find C_2 so that the velocity at this time is -120, or:

$$-120 = -32\frac{\sqrt{C_2}}{4} = -8\sqrt{C_2} \quad \Rightarrow \quad C_2 = 15^2 = 225$$

74. In this problem, be careful about mixing units! The problem has one velocity given as miles per hour, but then deceleration is in feet and seconds. Since the question asks about how far the car goes before it stops, it will probably be best to use feet and seconds.

First we convert miles per hour into feet per second (don't worry about recalling these conversion factors for an exam/quiz):

$$\frac{50 \text{ miles}}{1 \text{ hour}} \cdot \frac{5280 \text{ feet}}{1 \text{ mile}} \cdot \frac{1 \text{ hour}}{3600 \text{ sec}} = \frac{220}{3} \text{ ft/sec}$$

Now we can state the problem: Our acceleration is -22, and our "initial velocity" is $220/3$. Find the time at which velocity is zero, then determine how far we've traveled.

$$v' = -22 \quad v(0) = 220/3 \qquad \Rightarrow \quad v(t) = -22t + \frac{220}{3}$$

so the velocity is zero at time $t = 10/3$. Now we can determine displacement: $s(10/3) - s(0)$:

$$s(t) = -11t^2 + \frac{220}{3}t + C$$

so

$$s(10/3) - s(0) = -11(10/3)^2 + (220/3)(10/3) = 1100/9 \text{ ft}$$

77. In this problem (like in 74), we have to be careful about mixing units. We're told that

$$v(0) = 100 \qquad a(t) = -k \quad \Rightarrow \quad v(t) = -kt + 100$$

The vehicle will therefore stop at time $-kt + 100 = 0$, or $t = 100/k$.

The position is

$$s(t) = -\frac{1}{2}k^2t + 100t + C$$

If we assume $S(0) = 0 = C$, then wanting to stop within 80 meters means (in km):

$$s(100/k) = 80 \quad \Rightarrow \quad -\frac{k}{2}\left(\frac{100}{k}\right)^2 + 100 \cdot \frac{100}{k} = 0.08$$

Solving for k, we get

$$k = \frac{100^3}{16} = 62,500\text{km/hr}^2$$

If we want to convert this into meters and seconds, the answer would be

$$\frac{62500 \text{ km}}{1 \text{ hr}^2} \cdot \frac{1000 \text{ m}}{1 \text{ km}} \cdot \frac{1 \text{ hr}}{3600 \text{ sec}} \frac{1 \text{ hr}}{3600 \text{ sec}} \approx 4.82\text{m/s}^2$$