Homework hints, 5.5

9. Hint: Let $u = 1 - 2x$, $du = -2\,dx$.

13. Hint: Let $u = 5 - 3x$, $du = -3\,dx$.

16. Hint: Let $u = e^x$, $du = e^x\,dx$.

21. Hint: Let $u = \ln(x)$, so $du = \frac{1}{x}\,dx$

27. Hint: Let $u = x^3 + 3x$, so $du = (3x^2 + 3)\,dx = 3(x^2 + 1)\,dx$.

32. Hint: Let $u = \ln(x)$, so $du = \frac{1}{x}\,dx$.

33. Hint: Let $u = \sin(x)$, so $du = \cos(x)\,dx$. (The integrand becomes $\csc^2(u)$)

43. Hint: Let $u = \sin^{-1}(x)$, so $du = \frac{1}{\sqrt{1-x^2}}\,dx$.

44. This one is a bit trickier. While we don't know the antiderivative of $1/(1 + x^4)$, we do know that the derivative of $\tan^{-1}(x)$ is $1/(1 + x^2)$. So let $u = x^2$, and $du = 2x\,dx$. The remaining steps are then:

$$\int \frac{x}{1 + x^4}\,dx = \frac{1}{2}\int \frac{du}{1 + u^2} = \frac{1}{2}\tan^{-1}(u) + C = \frac{1}{2}\tan^{-1}(x^2) + C$$

45. It's probably easiest to split up the fraction first:

$$\int \frac{1 + x}{1 + x^2}\,dx = \int \frac{1}{1 + x^2}\,dx + \int \frac{x}{1 + x^2}\,dx$$

For the first integral, we can antidifferentiate directly, for the second integral, let $u = 1 + x^2$, so that $du = 2x\,dx$, etc.

For 53-55, I like to leave in terms of u by switching the integral bounds.

53. Let $u = \frac{\pi t}{2}$, so $du = \frac{\pi}{2}\,dt$, and:

$$\int_0^1 \cos(\pi t/2)\,dt = \frac{2}{\pi}\int_0^{\pi/2} \cos(u)\,du = \frac{2}{\pi}\sin(u)\Big|_0^{\pi/2} = \frac{2}{\pi}(1 - 0) = \frac{2}{\pi}$$

54. Similar, let $u = 3t - 1$ so that the integral becomes:

$$\int_0^1 (3t - 1)^{50}\,dt = \frac{1}{3}\int_{-1}^2 u^{50}\,du = \frac{2^{51} + 1}{3 \cdot 51}$$

55. Let $u = 1 + 7x$, so that the integral becomes:

$$\int_0^1 \sqrt[3]{1 + 7x}\,dx = \frac{1}{7}\int_1^8 u^{1/3}\,du = \frac{3}{28}u^{4/3}\Big|_1^8 = \frac{3}{28}(16 - 1) = \frac{45}{28}$$

60. Let $u = -x^2$, so $du = -2x\,dx$ and the integral becomes:

$$\int_0^1 x\mathrm{e}^{-x^2}\,dx = -\frac{1}{2}\int_0^{-1} \mathrm{e}^u\,du = \frac{1}{2}\int_{-1}^0 \mathrm{e}^u\,du = \frac{1}{2}(1-(1/e))$$

67. For this problem, we end up with an extra x. That is, if we let $u = x-1$, then $du = dx$. To substitute something for x, we take $x = u+1$ so that the integral becomes:

$$\int_1^2 x\sqrt{x-1}\,dx = \int_0^1 (u+1)u^{1/2}\,du = \int_0^1 u^{3/2} + u^{1/2}\,du = ...$$

82. Hint: If $P(t)$ is the population at time t, then

$$P(3) - P(0) = \int_0^3 r(t)\,dt$$

Then do the usual integration with $u = 1.12567t$.

83. The volume will simply be the antiderivative:

$$V(t) = \int_0^t \frac{1}{2}\sin(2\pi x/5)\,dx$$

85. To evaluate $\int_0^2 f(2x)\,dx$, let $u = 2x$ so that $du = 2\,dx$. The integral bounds are determined: If $x = 0$, then $u = 0$, and if $x = 2$, then $u = 4$, and:

$$\int_0^2 f(2x)\,dx = \int_0^4 f(u)\,du$$