Exam 1 Review (Spring 2015)

The exam will cover sections 4.9 (antiderivatives), 5.1-5.5 and Appendix E. The following questions are not meant to be exhaustive, so you should also be sure you've looked over your old quizzes and understand the homework. As you go through the material, remember that the quizzes were coming directly from the homework, and so on the exam, the questions may be more general.

Primary Theorems

FTC, Net Change Theorem, Integration by Substitution.

Riemann Sums

In this portion of the course, we introduced the summation notation (Appendix E), and defined the Riemann sum. We should be able to convert an integral into its corresponding Riemann sum, and vice versa. We should be able to compute the limit of the Riemann sum using the properties of the sum, the limit, and the sum formulas- You should know the formula for $\sum i$, and I will provide the formulas for $\sum i^2$ and $\sum i^3$ if you need them.

Evaluate the Integral

For this portion of the course, there are several methods for evaluating the integral that are handy to keep in mind:

- Integrate using geometry (area)
- Integrate using the table (that we've memorized)
- Simplify first, then integrate.
- u, du substitution.
- Integrate using symmetry.

Other Notes

For the sake of time, often on exam questions referring to area, I will ask you to *set up, but do not evaluate* the integral(s) you would need to compute to find the area of a given region (graphs of things more complicated than a parabola or line would be provided). Be sure you're reading the question carefully so that you don't spend a lot of time doing unnecessary computations.

You should memorize the table of integrals from 5.4 with the exception of the hyperbolic trig functions ($\sinh(x)$ and $\cosh(x)$), which we did not cover.

Here are your review questions. The first two questions are easiest if we have the theorem and definition memorized.

Exam 1 Review Questions

1. State the Fundamental Theorem of Calculus. I've started it below for you:

   Let f be continuous on $[a, b]$, and $g(x) = \int_a^x f(t)\,dt$ for $a \leq x \leq b$. Then: (there are two conclusions)

2. Give the *definition* of the definite integral (using right endpoints): $\int_a^b f(x)\,dx =?$

3. True or False (and give a short reason):

   (a) $\int_0^2 (x - x^3)\,dx$ represents the area under the curve $y = x - x^3$ from 0 to 2.

   (b) If $3 \leq f(x) \leq 5$ for all x, then $6 \leq \int_1^3 f(x)\,dx \leq 10$

   (c) If f, g are continuous and $f(x) \geq g(x)$ for all $a < x < b$, then $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$.

   (d) If $f(x) \geq g(x)$ for all $a < x < b$, then $f'(x) \geq g'(x)$ for $a < x < b$.

   (e) All continuous functions have derivatives.

   (f) All continuous functions have antiderivatives.

   (g) If f has a discontinuity at $x = 0$, then $\int_{-1}^1 f(x)\,dx$ does not exist.

4. The speed of a runner increased steadily during the first part of a race. Her speed (in feet per sec) at half-second intervals is given in the table. Give an upper and lower estimate for the distance she has run during these two seconds:

t	0	0.5	1.0	1.5	2.0
$v(t)$	0	3	6	7	10

5. Evaluate the integral, if it exists

   (a) $\int_1^9 \frac{\sqrt{u} - 2u^2}{u}\,du$

   (b) $\int 3^x + \frac{1}{x} + \sec^2(x)\,dx$

   (c) $\int_{-\pi/4}^{\pi/4} \frac{t^4 \tan(t)}{2 + \cos(t)}\,dt$

   (d) $\int_0^3 |x^2 - 4|\,dx$

   (e) $\int \frac{\cos(\ln(x))}{x}\,dx$

   (f) $\int_0^2 \sqrt{4 - x^2}\,dx$

   (g) $\int \frac{1}{\sqrt{1 - x^2}}\,dx$

   (h) $\int_{-2}^2 \frac{1}{x}\,dx$

   (i) $\int_0^1 (\sqrt[4]{w} + 1)^2\,dw$

   (j) $\int_{-2}^{-1} \frac{1}{x}\,dx$

   (k) $\int_0^{1/2} \frac{\sin^{-1}(x)}{\sqrt{1 - x^2}}\,dt$

   (l) $\int (1 + \tan(t))\sec^2(t)\,dt$

(m) $\int \tan(x)\,dx$

(n) $\int x\sqrt{1+x}\,dx$

(o) $\int \frac{y-1}{\sqrt{3y^2-6y+4}}\,dy$

(p) $\int_1^2 \frac{e^{1/x}}{x^2}\,dx$

(q) $\int_1^2 x\sqrt{x-1}\,dx$

(r) $\int_0^1 \frac{1}{(1+\sqrt{x})^4}\,dx$

6. Find the derivative of the function:

(a) $F(x) = \int_0^{x^2} \frac{\sqrt{t}}{1+t^2}\,dt$

(b) $y = \int_{\sqrt{x}}^{3x} \frac{e^t}{t}\,dt$

7. Evaluate the following by first recognizing it as a Riemann sum (then use the associated definite integral):

(a)

$$\lim_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^9 + \left(\frac{2}{n}\right)^9 + \left(\frac{3}{n}\right)^9 + \cdots \left(\frac{n}{n}\right)^9\right]$$

(b)

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}\left(5+\frac{2i}{n}\right)^4$$

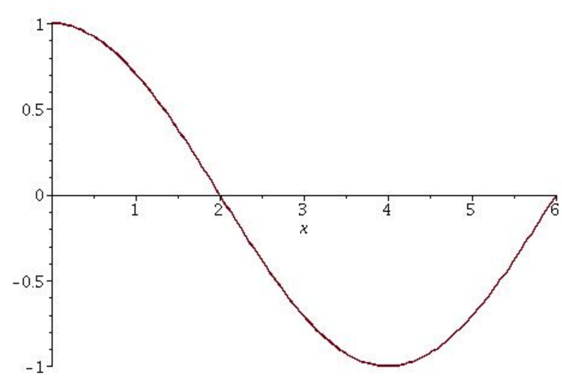
8. Evaluate the following definite integral via the definition (that is, using the Riemann sum).

$$\int_1^4 x^2 - 4x + 2\,dx$$

NOTE: $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

9. If $F(x) = \int_2^x f(t)\,dt$, where f is given below, which of the following is the smallest, and which is the largest?

(a) $F(0)$ (b) $F(1)$ (c) $F(2)$ (d) $F(3)$ (e) $F(4)$



10. Continuing with the function F in Problem 9, at what values of x does F have local maximum and minimum values? On what interval(s) is F concave downward?

11. Prove the following using Riemann sums (for credit, you must use the Riemann sum):

$$\int_a^b x\,dx = \frac{b^2 - a^2}{2}$$

12. Evaluate:

    (a) $\displaystyle\int_0^1 \frac{d}{dx}\left(e^{\tan^{-1}(x)}\right)dx$

    (b) $\displaystyle\frac{d}{dx}\int_0^1 e^{\tan^{-1}(x)}\,dx$

    (c) $\displaystyle\frac{d}{dx}\int_0^x e^{\tan^{-1}(t)}\,dt$

13. If $f(x) = \displaystyle\int_0^{\sin(x)} \sqrt{1+t^2}\,dt$ and $g(y) = \displaystyle\int_3^y f(x)\,dx$, then find $g''(\pi/6)$.

14. A particle moves along a line with velocity $v(t) = t^2 - t$, where v is measured in meters per second. Find (a) the displacement and (b) the distance traveled by the particle during the time interval $[0, 5]$.

15. A bacteria population is 2500 at time $t = 0$ and its rate of growth is $1000 \cdot 2^t$ bacteria per hour after t hours. What is the population after 1 hour?

16. If f is continuous and $\displaystyle\int_0^9 f(x)\,dx = 4$, find $\displaystyle\int_0^3 xf(x^2)\,dx$

17. If $f''(x) = 2 - 12x$, $f(0) = 0$ and $f(2) = 15$, find $f(x)$.

18. Evaluate the following integral using geometry (and area). You might use a property of integrals first.

$$\int_0^1 x + \sqrt{1-x^2}\,dx$$

19. Given the following sum, first express it using sigma notation, then find the limit.

$$\lim_{n\to\infty} \frac{3}{n}\left[\left(\frac{1}{n^2}+1\right) + \left(\frac{4}{n^2}+1\right) + \left(\frac{9}{n^2}+1\right) + \cdots \left(\frac{n^2}{n^2}+1\right)\right]$$

20. Find four different integrals that are represented by the following Riemann sum: $\displaystyle\sum_{i=1}^{n}\left(\frac{3}{n}\right)\left[4 - \frac{9i^2}{n^2}\right]$