Exam 2: General Notes

This exam will cover material from 6.1-6.3 and 7.1-7.5. No calculators will be allowed, but you will be given the table given below.

Please be sure to read the question carefully to see how far into the integral you will need to go. Sometimes I will ask you only to do the set up, sometimes I will ask you to only go as far as the substitution, sometimes you will need to evaluate the integral completely.

As usual, these questions are not meant to be exhaustive, but are meant to give you a better sense of what to expect on the exam. You should also go through the old quizzes and homework.

Review Questions

1. Find the area bounded between the regions $y = 1 - 2x^2$ and $y = |x|$.

2. Evaluate the integral and interpret it as the area of a region (sketch it).

$$\int_0^4 |\sqrt{x+2} - x|\, dx$$

3. Let R be the region in the first quadrant bounded by $y = x^3$ and $y = 2x - x^2$. Calculate the following quantities: (Exam note: Region R would typically be plotted for you).

   (a) The area of R.

   (b) Volume obtained by rotating R about the $x-$axis.

   (c) Volume obtained by rotating R about the $y-$axis.

4. Use any method to find an integral representing the volume generated by rotating the given region about the specified axis. You do NOT need to evaluate the integral:

   (a) $y = \sqrt{x}, y = 0, x = 1$; about $x = 2$.

   (b) $y = x^2$, $y = 2 - x^2$; about $x = 1$.

   (c) $y = x^2$, $y = 2 - x^2$; about $y = -3$.

   (d) $y = \tan(x)$, $y = x$, $x = \pi/3$; about the $y-$axis.

5. Write the partial fraction decomposition for each of the following (do not actually solve for the coefficients):

   (a) $\dfrac{3 - 4x^2}{(2x+1)^3}$

   (b) $\dfrac{7x - 41}{(x-1)^2(2-x)}$

   (c) $\dfrac{x+1}{x^3(x^2 - x + 10)^2}$

6. Integrate the following:
$$\int \frac{2x^3 - x^2 - 4x - 13}{x^2 - x - 2}\, dx$$

7. If $x = \tan(\theta)$, show that $\sin(2\theta) = \frac{2x}{1+x^2}$.

8. True or False? (And give a short reason)

   (a) To find $\int \sin^2(x)\cos^5(x)\, dx$, rewrite the integrand as $\sin^2(x)(1-\sin^2(x))^2\cos(x)$.

   (b) Integration by parts is the integral version of the Product Rule for derivatives.

   (c) To find $\int \frac{2x-3}{x^2-3x+5}\, dx$, start by completing the square in the denominator.

   (d) To find $\int \frac{3}{x^2-3x+5}\, dx$, start by completing the square in the denominator.

   (e) To find $\int \frac{3}{x^2-4x+3}\, dx$, start by completing the square in the denominator.

9. Evaluate using any method, unless specified below:

   (a) $\int \frac{4\, dx}{(4+x^2)^{3/2}}$

   (b) $\int \tan^3(x)\sec^2(x)\, dx$

   (c) $\int \frac{3x+2}{x^2+6x+8}$

   (d) $\int \frac{t^2\cos(t^3-2)}{\sin^2(t^3-2)}\, dt$

   (e) $\int \cos^5(x)\sqrt{\sin(x)}\, dx$

   (f) $\int \frac{x}{x^2+4}\, dx$

   (g) $\int \frac{dx}{\sqrt{1-6x-x^2}}$

   (h) $\int \frac{x-1}{x^2+3}\, dx$

   (i) $\int \sin^2(3t)\, dt$

   (j) $\int \frac{3x-2}{(x^2+2)^2}$

   (k) $\int \sin^{-1}(x)\, dx$

   (l) $\int x^3\sqrt{x^2+4}\, dx$

   (m) $\int \sqrt{2x-x^2}\, dx$

   (n) $\int \sqrt{t}\ln(t)\, dt$

   (o) $\int \frac{3x-1}{(x+2)(x-3)}\, dx$

   (p) $\int \ln(y^2+9)\, dy$

   (q) $\int \frac{\sin^3(x)}{\cos^4(x)}\, dx$

   (r) $\int e^{-x}\sin(2x)\, dx$

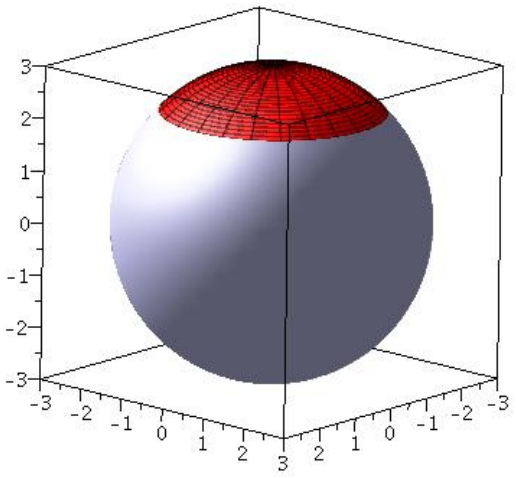
   (s) $\int \frac{w}{\sqrt{w+5}}\, dw$

   (t) $\int y^2 e^{-3y}\, dy$

   (u) $\int_0^1 \frac{dx}{(x^2+1)^2}$

   (v) $\int_0^3 \frac{x}{\sqrt{36-x^2}}\, dx$

10. Suppose we have a sphere with radius 3. Set up an integral representing the volume of the cap of the sphere that comes down 1 unit.



11. Simplify the following expressions (using a triangle):

$$\tan(\sin^{-1}(x/2)) \qquad \cos(\tan^{-1}(2/x))$$

12. If $\sin(\theta) = \frac{x}{\sqrt{x^2+4}}$, find an expression for $\sin(2\theta)$ and $\cos(2\theta)$.

13. Suppose that:

x	$f(x)$	$f'(x)$
1	2	5
4	7	3

where f'' is continuous. Find the value of $\int_1^4 xf''(x)\,dx$.

14. Complete the square:

    (a) $2x^2 + 8x$

    (b) $6 - 3x^2$

15. Perform the long division shown. Write the result as we would if we were starting Partial Fraction Decomposition.

$$\frac{x^4 - 4x^3 - 4x^2 - 5x + 4}{x^2 + x + 1}$$