Solutions to Exponential and Logarithmic Functions (part 4)

Notes: The main rule we'll use to compute things with $\log_a$ is:

$$\log_a(b) = \frac{\ln(b)}{\ln(a)}$$

4. Differentiate, or integrate, as shown:

(a) Chain Rule, outside function is 3^x, whose derivative is $\ln(3) \cdot 3^x$, inside function is $x^2 + \sin(x)$.

$$\frac{\mathrm{d}}{\mathrm{dx}}\, 3^{x^2+\sin(x)} = \ln(3) \cdot 3^{x^2+\sin(x)} \cdot (2x + \cos(x))$$

(b)

$$\frac{1}{\ln(5)} \frac{\mathrm{d}}{\mathrm{dx}}\, \ln(x^2 + \mathrm{e}^{3x}) = \frac{1}{5} \frac{1}{x^2 + \mathrm{e}^{3x}} \cdot (2x + 3\mathrm{e}^{3x})$$

(c) (I'll use the product rule, but you could also use the quotient rule) $\frac{\mathrm{d}}{\mathrm{dx}}\, \log_x(3x+1) =$

$$\frac{\mathrm{d}}{\mathrm{dx}} \frac{\ln(3x+1)}{\ln(x)} = \frac{\mathrm{d}}{\mathrm{dx}}\,(\ln(3x+1))(\ln(x))^{-1} = \frac{3}{3x+1}(\ln(x))^{-1} + \ln(3x+1)(-(\ln(x))^{-2}\frac{1}{x})$$

We can simplify a bit:

$$\frac{3}{(3x+1)\ln(x)} - \frac{\ln(3x+1)}{x(\ln(x))^2}$$

(d) $\frac{\mathrm{d}}{\mathrm{dx}}\, \sin^{-1}(\mathrm{e}^x) =$

$$\frac{1}{1 + (\mathrm{e}^x)^2} = \frac{1}{1 + \mathrm{e}^{2x}}$$

(e) $\frac{\mathrm{d}}{\mathrm{dx}}\, \ln\left(\frac{1}{x}\right) + \frac{1}{\ln(x)} =$

$$\frac{1}{\frac{1}{x}} \cdot (-\frac{1}{x^2}) - (\ln(x))^{-2}\frac{1}{x}$$

which simplifies to

$$-\frac{1}{x} - \frac{1}{x(\ln(x))^2}$$

(f) $\displaystyle\int \frac{x^2+x+1}{x}\,\mathrm{dx} = \int x + 1 + \frac{1}{x}\, dx = \frac{1}{2}x^2 + x + \ln(x) + C$

(g) $\displaystyle\int \frac{x+1}{x^2+2x}\,\mathrm{dx} =$

Let $u = x^2 + 2x$, $du = (2x+2)dx = 2(x+1)dx$, so:

$$\frac{1}{2}\int \frac{1}{u}\,\mathrm{du} = \frac{1}{2}\ln(u) + C = \frac{1}{2}\ln(x^2+2x) + C$$

(h) $\displaystyle\int 5^t\,\mathrm{dt} = \frac{1}{\ln(5)}5^t + C$

(i) $\displaystyle\int \frac{1}{w\ln(w)}\,\mathrm{dw} =$

Let $u = \ln(w)$, so $du = \frac{1}{w}\, dw$. Then:

$$\int \frac{1}{u}\,\mathrm{du} = \ln(u) + C = \ln(\ln(w)) + C$$

(Double check by integrating!)