Final Exam Review Questions

This review sheet is provided to you so that you have lots of exercises that are presented out of context. To study, you should be comfortable solving these problems, and also look at old exams and homework (both turned in and not turned in problems). The solutions include the relevant section of the textbook, so if you have questions, you can refer back to it.

INSTRUCTIONS: Whenever we evaluate a definite integral, assume the instructions say: "Evaluate the integral, **if it exists**". When evaluating an indefinite integral, you may assume that the function is continuous on its domain.

1. Set up an integral for the volume of the solid obtained by rotating the region defined by $y = \sqrt{x-1}$, $y = 0$ and $x = 5$ about the $y-$axis.

2. Write the area under $y = \sqrt[3]{x}$, $0 \leq x \leq 8$ as the limit of a Riemann sum (use right endpoints).

3. Find the volume of the solid obtained by rotating the region bounded by: $y = \frac{1}{x}$, $y = 0$, $x = 1$, $x = 3$ about $y = -1$.

4. The integral $\pi \int_2^5 y\, dy$ represented the volume of a solid. Describe the solid.

5. Determine a region whose area is equal to the following limit (do not evaluate the limit): $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\sqrt{1 + \frac{3i}{n}}$

6. Write the following limit as a definite integral on the given interval: $\lim_{n\to\infty} \sum_{i=1}^{n} x_i \sin(x_i)\Delta x$, $[0, \pi]$.

7. Find the area between the curves $y^2 = x$ and $x - 2y = 3$.

8. Write the following difference as a single integral: $\int_2^{10} f(x)\, dx - \int_2^7 f(x)\, dx$

9. If $\int_0^1 f(x)dx = 2$, $\int_0^4 f(x)dx = -6$, and $\int_3^4 f(x)\, dx = 1$, find $\int_1^3 f(x)\, dx$.

10. If $\int_0^1 f(x)\, dx = \frac{1}{3}$, what is $\int_0^1 5 - 6f(x)\, dx$?

11. Compute $\frac{dF}{dx}$, if $F(x) = \int_x^2 \cos(t^2)\, dt$

12. Compute $\frac{dg}{dy}$, if $g(y) = \int_3^{\sqrt{y}} \frac{\cos(t)}{t}\, dt$.

13. Evaluate: $\int_{-1}^{1} 3t^{-4}\, dt$

14. Find $\frac{dy}{dx}$, if $y = \int_{\cos(x)}^{5x} cos(t^2)\, dt$

15. Evaluate the limit by recognizing the sum as a Riemann sum:

$$\lim_{n\to\infty} \frac{1}{n}\left(\sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \ldots + \sqrt{\frac{n}{n}}\right)$$

16. Write the following integral as the limit of a Riemann sum (use right endpoints): $\int_0^5 (1+2x^3)\,dx$

17. Given that $\int_4^9 \sqrt{x}\,dx = \frac{38}{3}$, what is $\int_9^4 \sqrt{t}\,dt$?

18. Let $f(x) = e^x$ on the interval $[0, 2]$. (a) Find the average value of f. (b) Find c such that $f_{\text{avg}} = c$.

19. The velocity function is $v(t) = 3t - 5$. (a) Find the displacement, (b) Find the distance traveled.

20. Exercise 7, pg. 427 (There are some graphs to consider).

21. Suppose $h(1) = -2$, $h'(1) = 2$, $h''(1) = 3$, $h(2) = 6$, $h'(2) = 5$, and $h''(2) = 13$, and h'' is continuous. Evaluate $\int_1^2 h''(u)\,du$.

22. Find the area between the curves $y = |x|$ and $y = x^2 - 2$.

23. If f is continuous and $\int_0^9 f(x)\,dx = 4$, find $\int_0^3 xf(x^2)\,dx$.

24. Problem 41, p. 439 (Pictures)

25. Find a so that half the area under the curve $y = \frac{1}{x^2}$ lies in the interval $[1, a]$ and half of the area lies in the interval $[a, 4]$.

26. Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by $y = x$, $y = 4x - x^2$, about $x = 7$.

27. Find the centroid of the region bounded by the curves $y = 4 - x^2$ and $y = x + 2$.

28. Review the Work problems, 17-20, p. 459.

29. If f is continuous, and $\int_1^3 f(x)dx = 8$, show that f take on the value 4 at least once in the interval $[1, 3]$.

30. Let R be the region in the first quadrant bounded by $y = x^3$ and $y = 2x - x^2$. Calculate: (a) The area of R, (b) Volume obtained by rotating R about the $x-$axis (c) Volume obtained by rotating R about the $y-$axis, (d) The centroid of R.

31. Exercises 13 and 14, p. 576 (Find the centroid).

32. Set up an integral to find the arc length of the curve $y = \frac{x^3}{6} + \frac{1}{2x}$, $1 \leq x \leq 2$.

33. What does the Theorem of Pappus say?

34. Evaluate: $\frac{d}{dx} \int_0^1 x^2 \sin(x)\,dx$

35. Find the centroid of the region bounded by the curves $y = x^2$ and $y = x$.

36. Write the appropriate partial fraction expansion for the following expression (do not solve for the constants):

$$\frac{1 + x}{(x - 1)^2(x^2 + 1)}$$

37. If an integral has $\sqrt{2x - x^2}$, what is the appropriate trigonometric substitution?

38. If an integral has $\sqrt{1 - 2(x+3)^2}$, what is the appropriate trigonometric substitution?

39. What is the derivative of $\sin^{-1}(x)$? $\tan^{-1}(x)$?

40. Let f be differentiable. Integrate by parts: $\int f(x)\, dx$.

41. Suppose you are integrating $P(x)/Q(x)$, where P and Q are polynomials. Explain the process by which we integrate this expression. (Consider the degrees of P and Q).

42. What was the Mean Value Theorem for Integrals?

43. True or False: Every elementary function has an antiderivative.

44. If we have $\int_a^b f(x)dx$, and $f(x)$ has a vertical asymptote at c, where c is in the interval $[a, b]$, explain how we proceed.

45. Evaluate: $\int_0^\infty t\mathrm{e}^{-st}\, dt$, where s is a positive constant.

46. What does L'Hospital's rule say?

47. Evaluate:

   (a) $\displaystyle\int \frac{\sec^2(x)}{1 - \tan(x)}\, dx$

   (b) $\displaystyle\int_{-1}^{1} \frac{x^5 + x^3 + x}{x^4 + x^2 + 1}\, dx$

   (c) $\displaystyle\int \tan^7 x \sec^3 x\, dx$

   (d) $\displaystyle\int \frac{dx}{x \ln(x)}$

   (e) $\displaystyle\int \frac{1}{y^2 - 4y - 12}\, dy$

   (f) $\displaystyle\int (1 - t)(2 + t^2)\, dt$

   (g) $\displaystyle\int u(\sqrt{u} + \sqrt[3]{u})\, du$

   (h) $\displaystyle\int_1^\infty \frac{1}{(2x+1)^3}\, dx$

   (i) $\displaystyle\int_{-1}^{2} |x - x^2|\, dx$

   (j) $\displaystyle\int_1^4 \frac{\mathrm{e}^{1/x}}{x^2}\, dx$

   (k) $\displaystyle\int \frac{x^2}{(4 - x^2)^{3/2}}\, dx$

   (l) $\displaystyle\int \frac{1}{1 + \mathrm{e}^x}\, dx$

(m) $\displaystyle\int \frac{\tan^{-1}(x)}{1+x^2}\ dx$

(n) $\displaystyle\int_{-a}^{a} x\sqrt{x^2-a^2}\ dx$

(o) $\displaystyle\int_0^2 \frac{dx}{(2x-3)^2}\ dx$

(p) $\displaystyle\int \sin^2 x \cos^5 x\ dx$

(q) $\displaystyle\int \frac{x}{x^2+1}\ dx$

(r) $\displaystyle\int \frac{1}{\sqrt{x^2-4x}}\ dx$

(s) $\displaystyle\int \sin(4x)\cos(3x)\ dx$

(t) $\displaystyle\int x^4 \ln(x)\ dx$